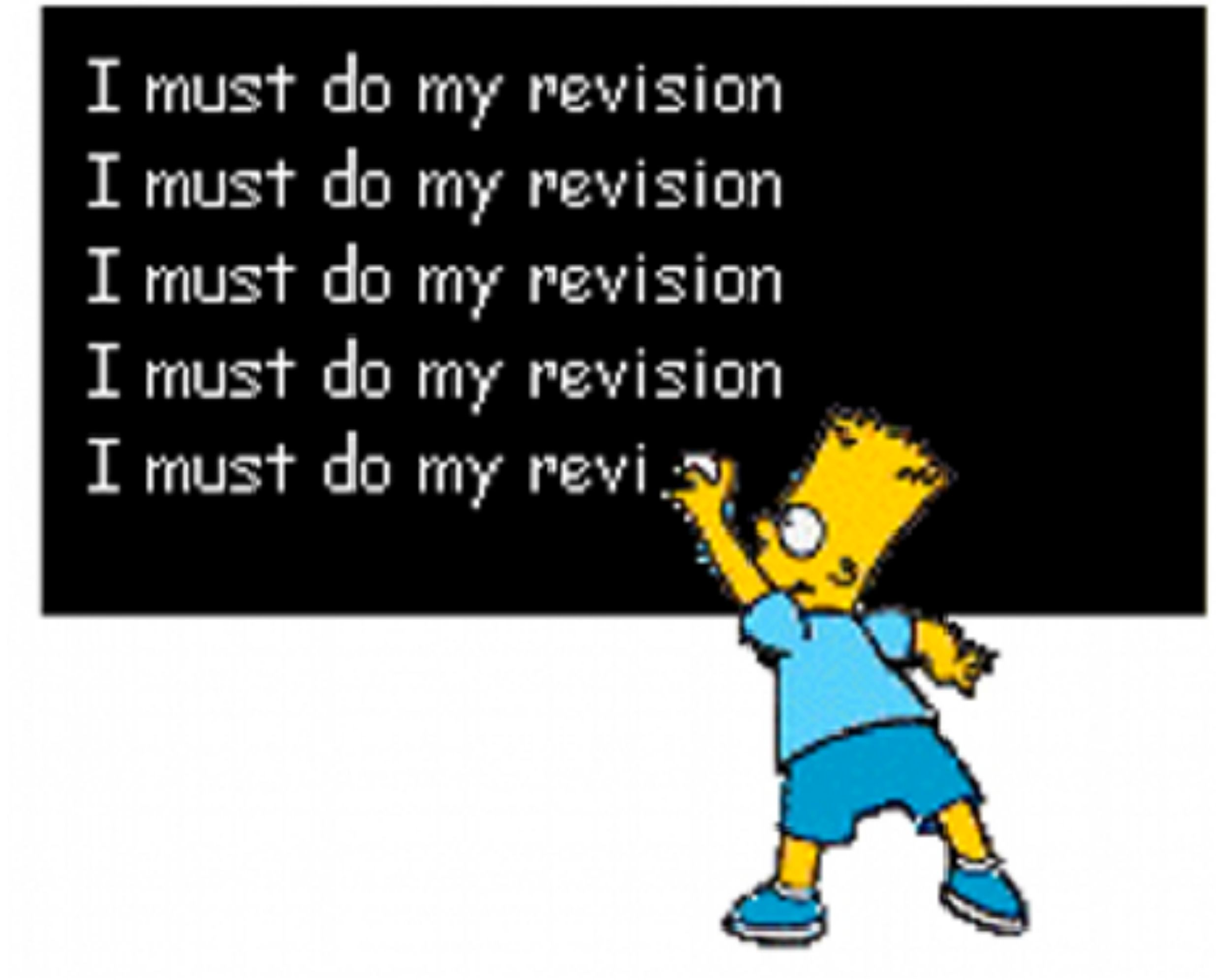
CO3091 - Computational Intelligence and Software Engineering

Lecture 27



Feature Selection and Revision of Optimisation Algorithms

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Image from: http://www.literaturedelights.com/wp-content/uploads/2016/03/revision-simpsons.jpg

Overview

- Brute-force search
- Hill-Climbing
- Simulated Annealing
- Evolutionary Algorithms
- Constraint Handling
- Multi-objective Evolutionary Algorithms
- What would be a good algorithm / design for feature selection?

Doodle Poll

- The doodle poll to sign up for an optional lab session this Friday will be released this afternoon.

Brute-Force Search

- Brute-force search = exhaustive search = generate and test.
- Systematically enumerate all possible candidates for the solution and check which one is the best.
- What are the advantages and disadvantages of brute force?
- Guaranteed to find the optimal solution.
- Can we use brute-force search to solve optimisation problems?

Image from: http://vignette2.wikia.nocookie.net/creepypasta/images/8/80/Creepycemetery.jpg/revision/latest?cb=20131013135547

Problem: high computational complexity.

Computational Intelligence

- No single definition.
- Heuristic algorithms, which aim to find good solutions to problems in a reasonable amount of time.*
  - Typically not guaranteed to find the optimum, but able to find near-optimal solutions.
  - What type of problems could benefit from heuristic algorithms?
    - Problems where we cannot afford enumerating all possible solutions to guarantee optimality.
    - Problems where no exact algorithm exists to solve the problem in polinomial time.
    - Problems where a near-optimal solution is acceptable.
- Learning algorithms that can build models based on data.

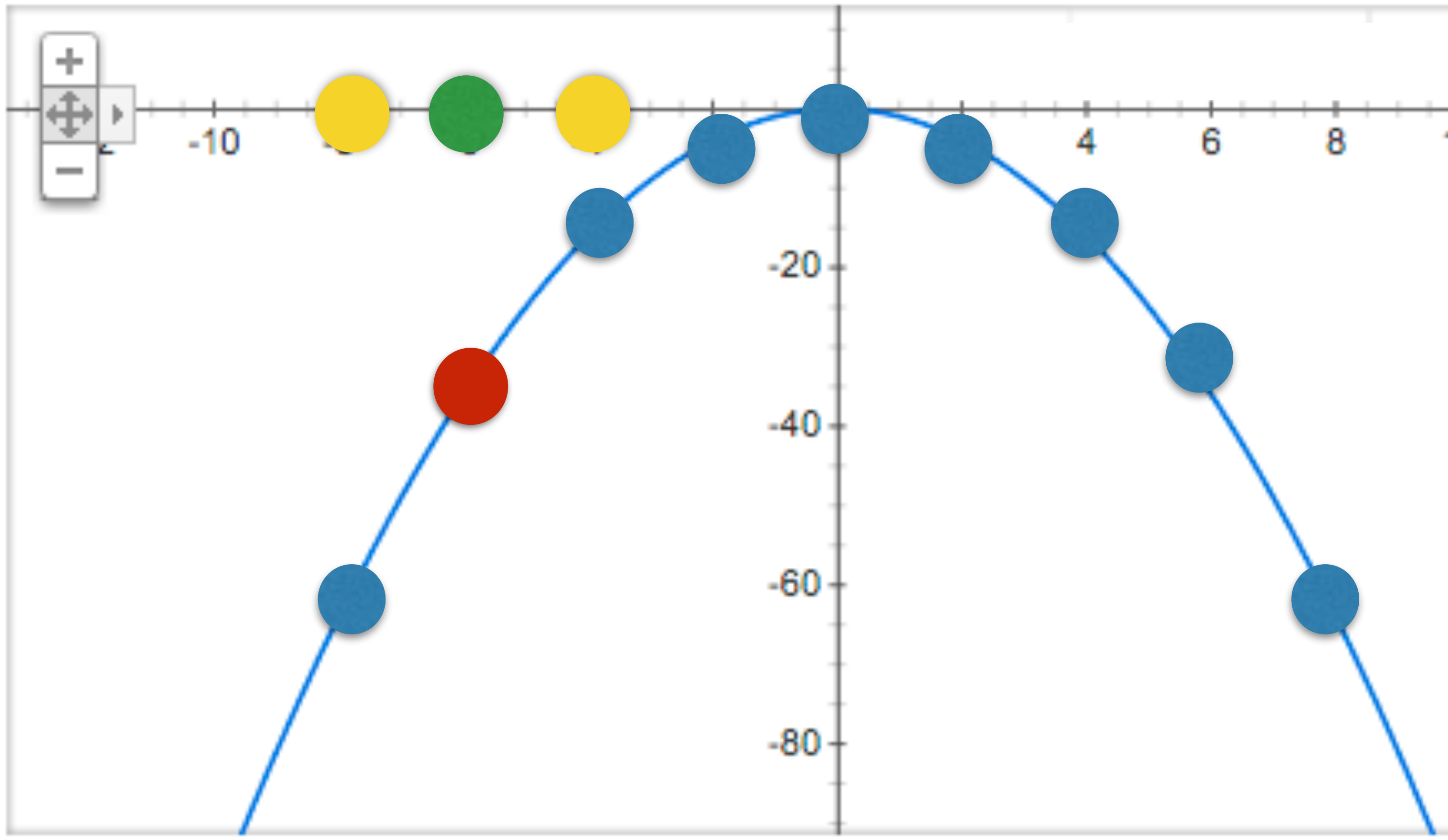
* http://ukci.cs.manchester.ac.uk/intro.html

Hill-Climbing — Illustrative Example

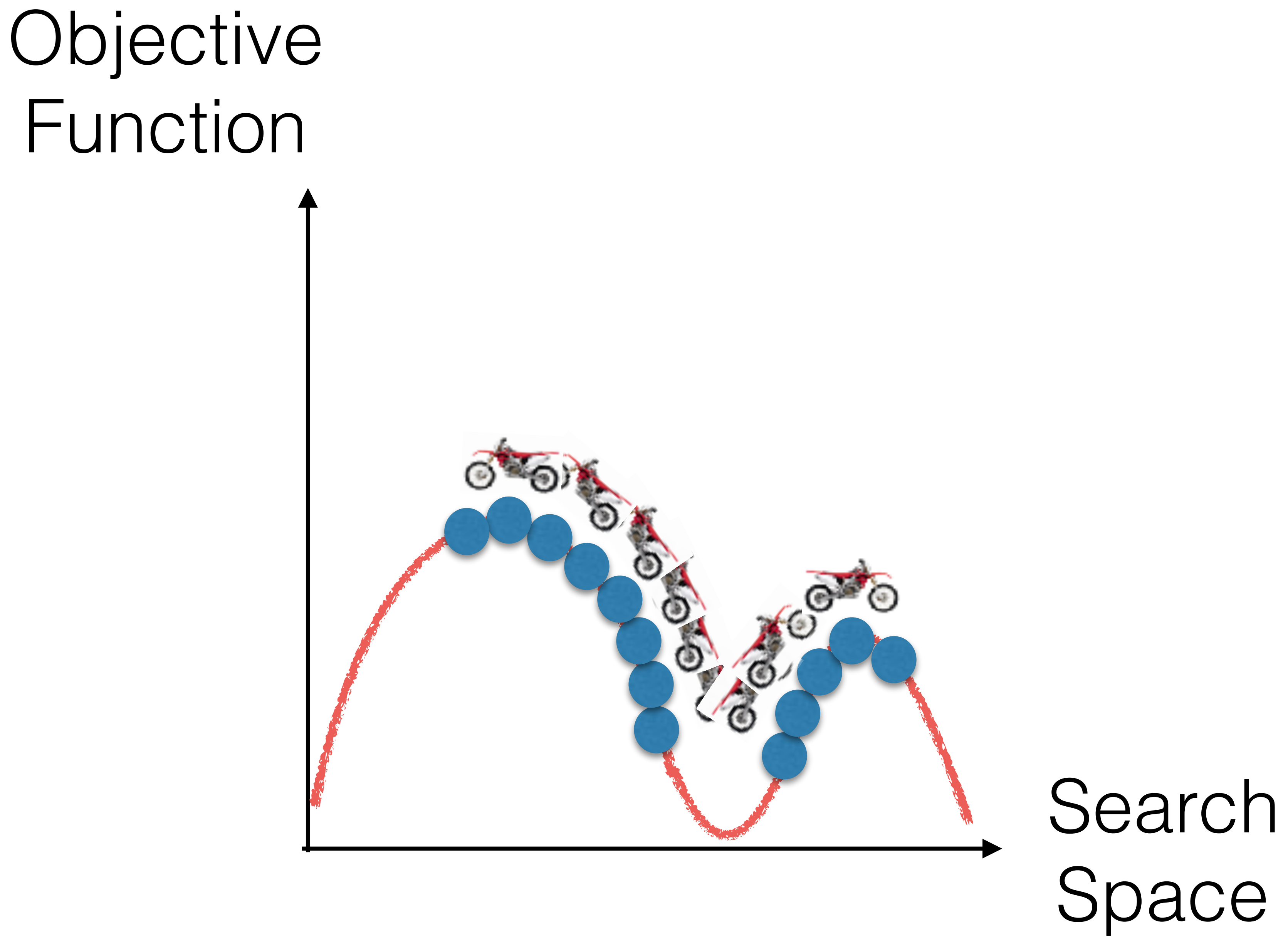
Hill-Climbing (assuming maximisation)

```
1. current_solution = generate initial solution randomly

2. Repeat:

    2.1 generate neighbour solutions (differ from current solution by a single element)

    2.2 best_neighbour = get highest quality neighbour of current_solution

    2.3 If quality(best_neighbour) <= quality(current_solution)

        2.3.1 Return current_solution

    2.4 current_solution = best_neighbour
```

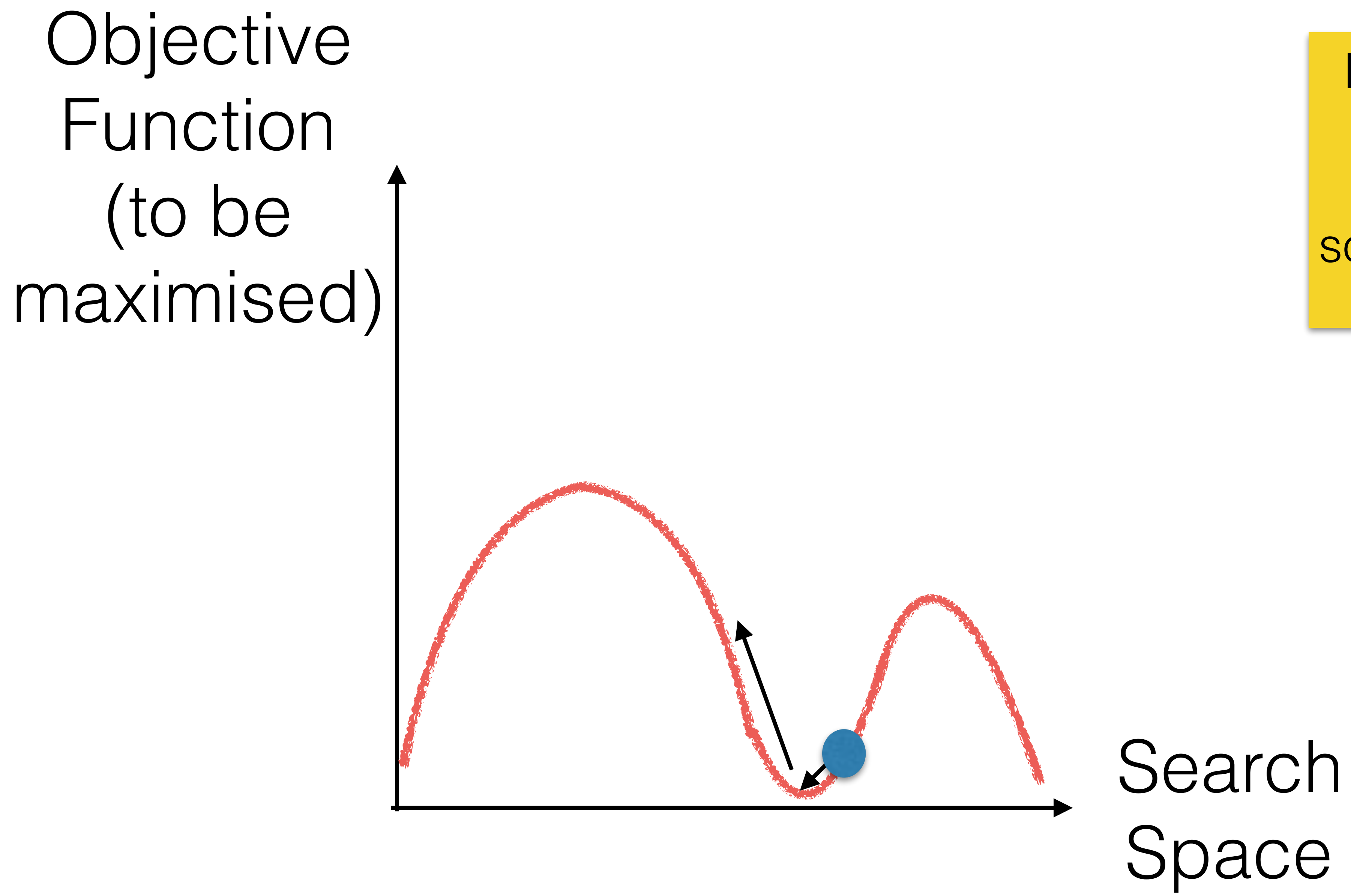
maximise $f(x) = -x^2$



Hill-Climbing — General Idea



Simulated Annealing — Motivation



If we could sometimes accept a downward move, we would have some chance to move to another hill.

In simulated annealing, instead of taking the best neighbour, we pick a random neighbour and give some chance to accept it, even if its quality is worse than that of the current solution.

Metallurgy Annealing

- A blacksmith heats the metal to a

High temperature = high probability

molecules can move fast and randomly.

Image from: http://www.stormthecastle.com/indeximages/sting-steel-thumb.jpg

- The blacksmith then lets it cool down slowly.

Low temperature = low probability

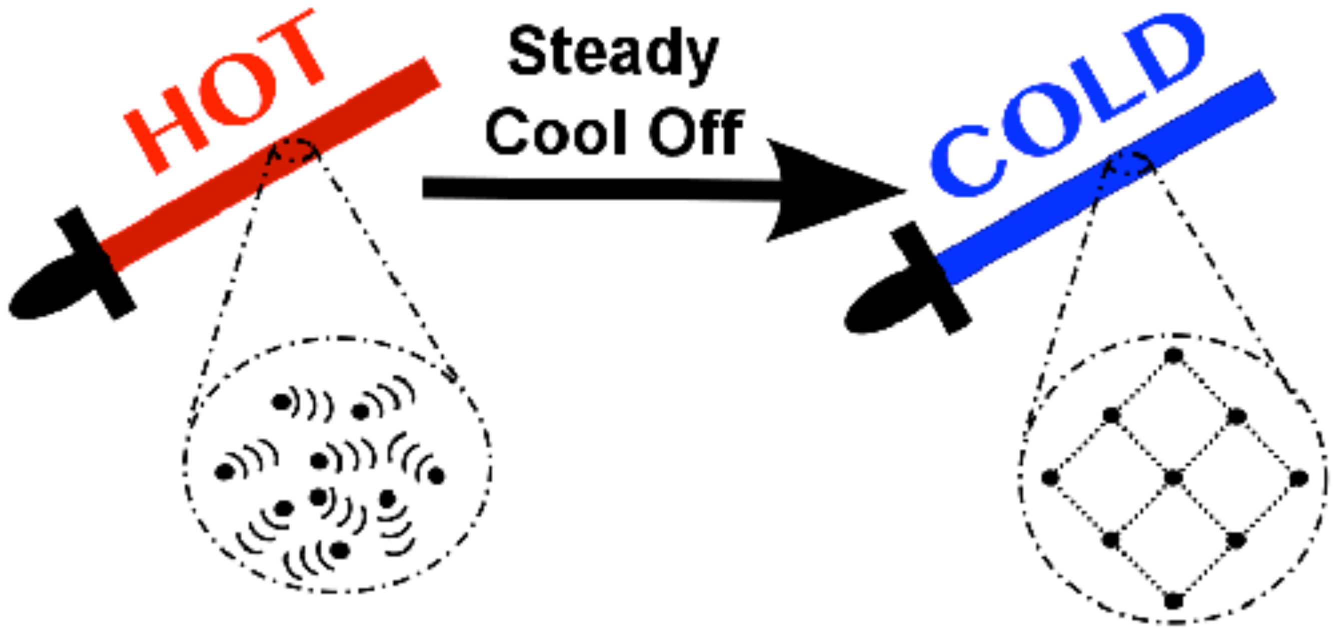
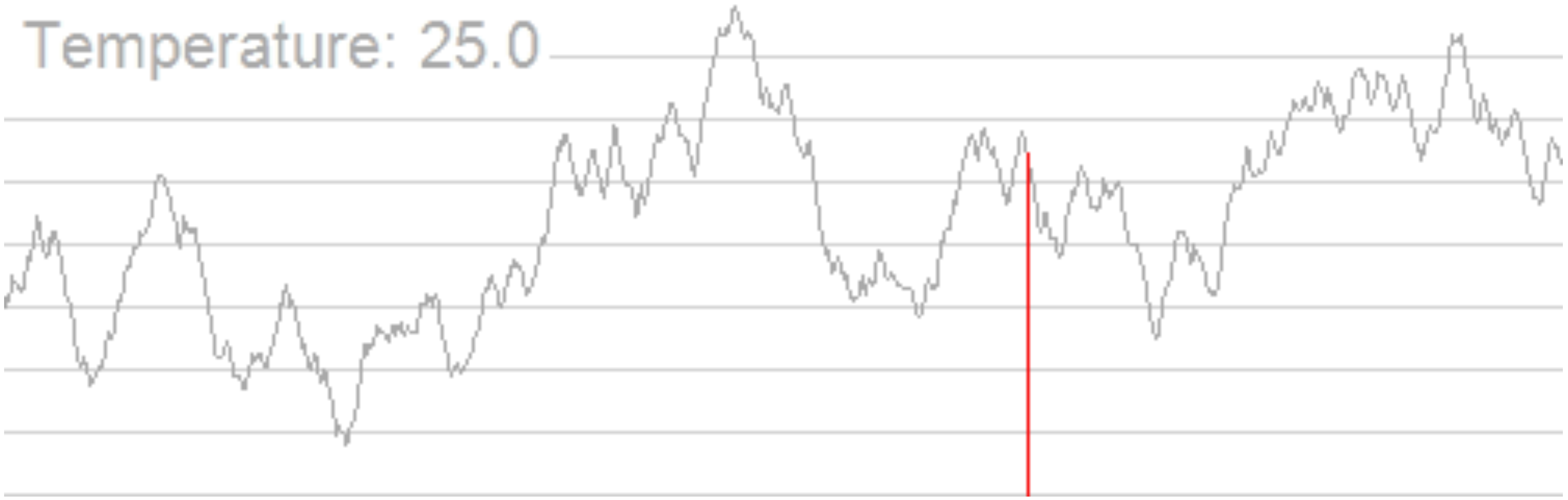


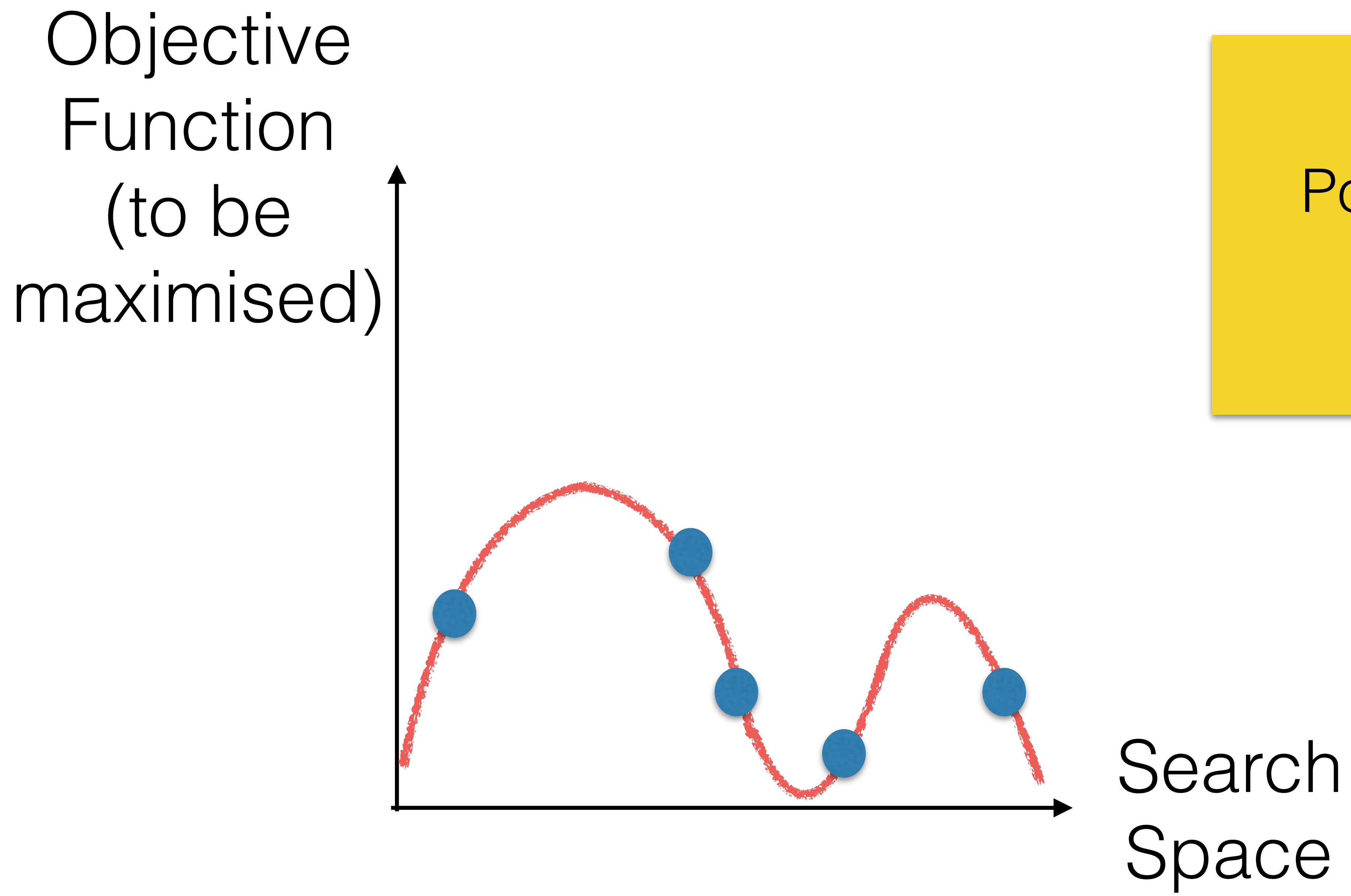
Image from: http://2.bp.blogspot.com/--kOlrodykkg/UbfVZ0_I5HI/AAAAAAAAAJ4/0rQ98g6tDDA/s1600/annealingAtoms.png

- This makes the sword stronger than the untreated steel.



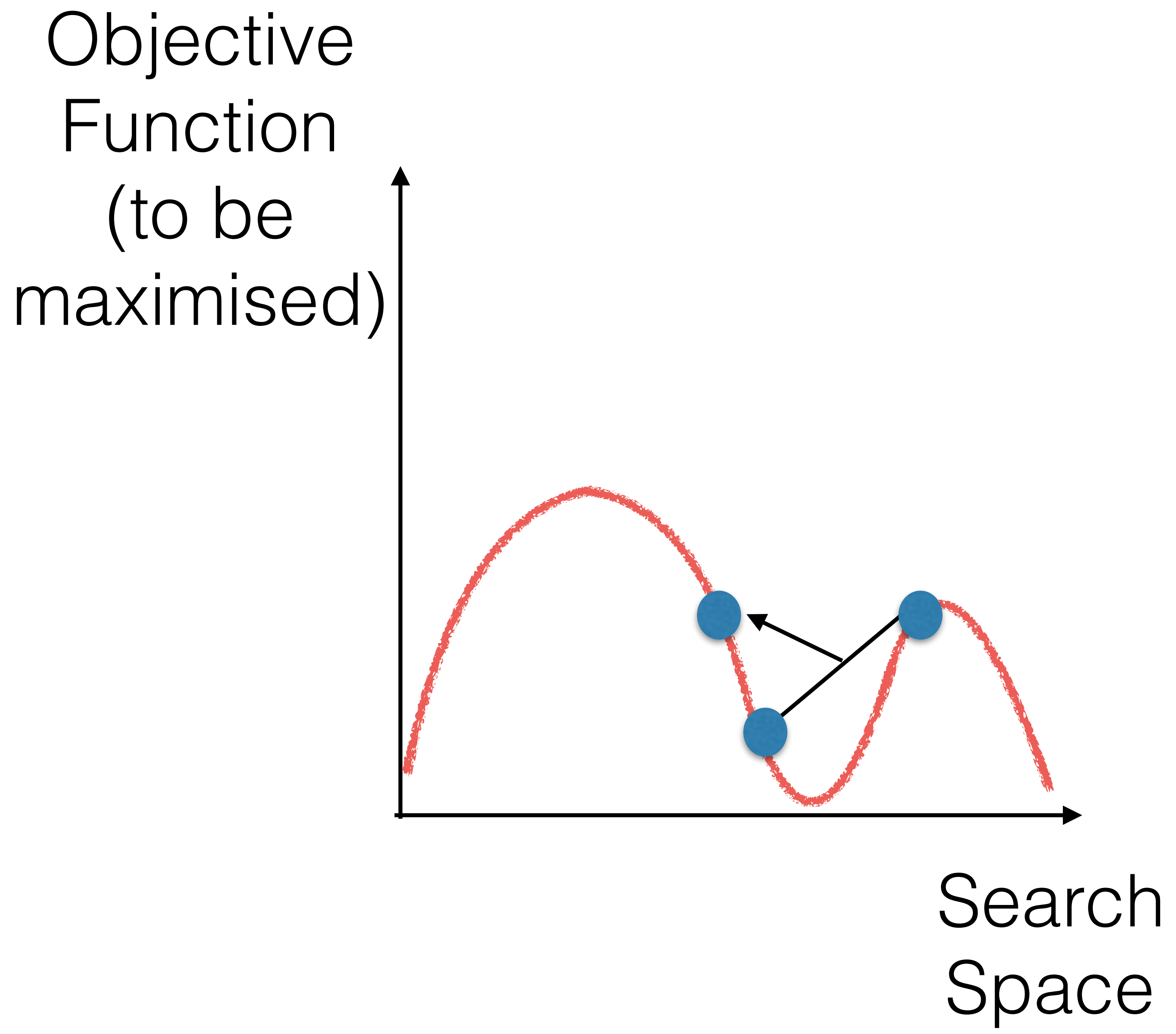
[By Kingpin13 - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=25010763]

Evolutionary Algorithms (EAs)



Population helps exploration.

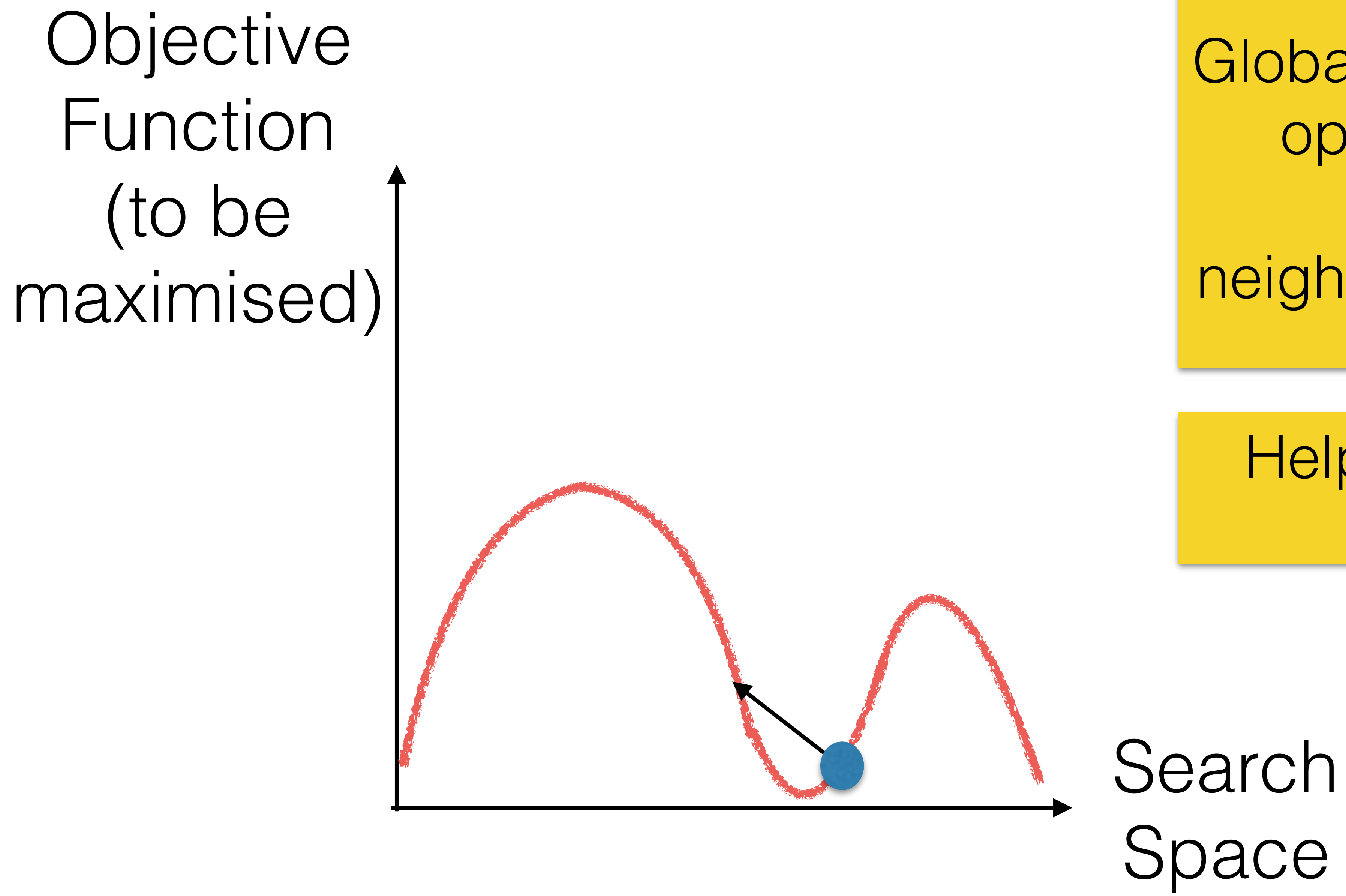
Evolutionary Algorithms (EAs)



Bad individuals have some (small) chance to reproduce.

Helps to avoid local optima.

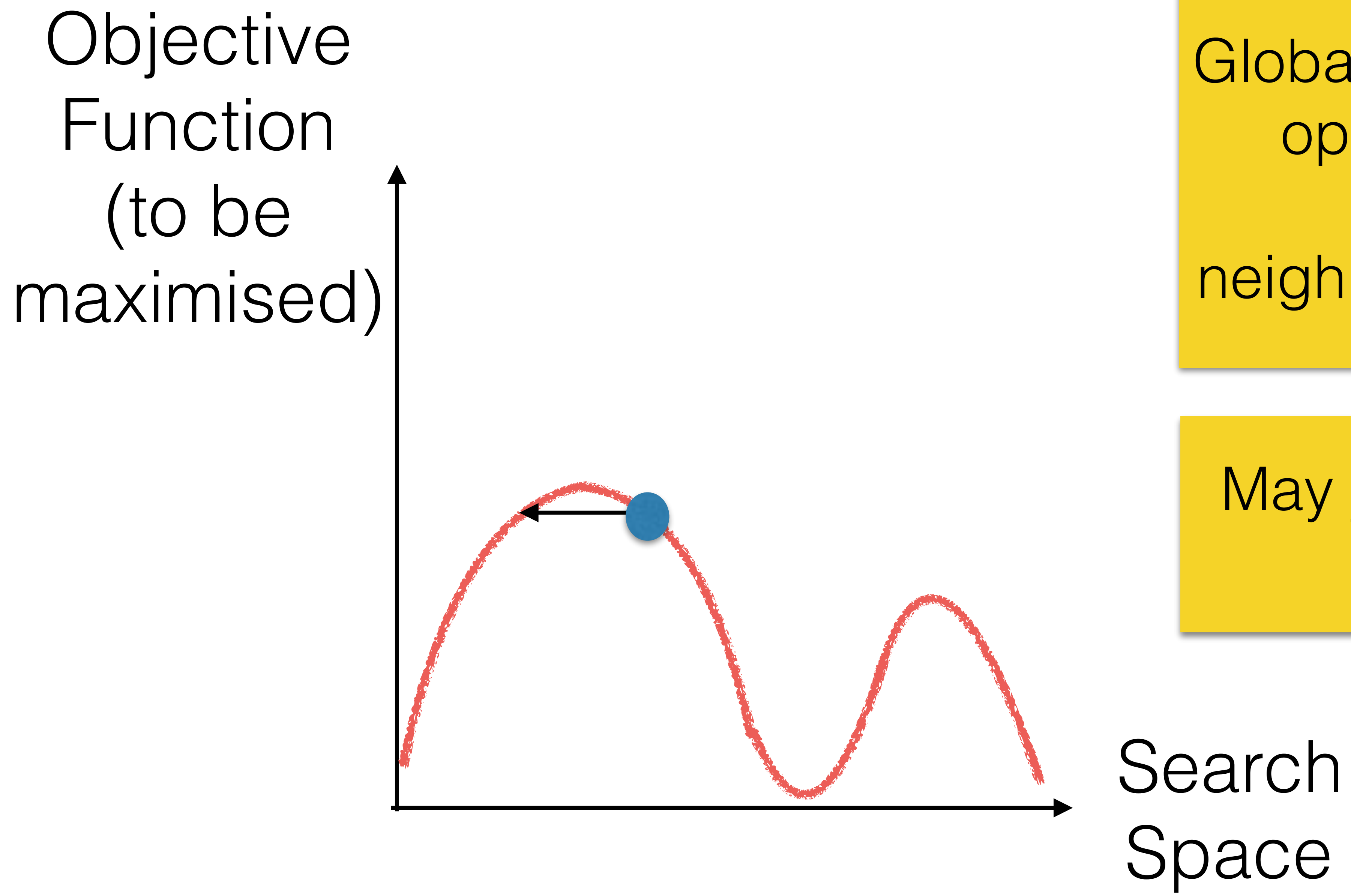
Evolutionary Algorithms (EAs)



Global search: variation operators are not restricted to neighbouring solutions.

Helps to avoid local optima.

Evolutionary Algorithms (EAs)



Global search: variation operators are not restricted to neighbouring solutions.

May jump passed the optimum.

EA's Pseudocode

```
Evolutionary Algorithm
  1. Initialise population
  2. Evaluate each individual (determine their fitness)
  3. Repeat (until a termination condition is satisfied)
      3.1 Select parents
      3.2 Recombine parents with probability Pc
      3.3 Mutate resulting offspring with probability Pm
      3.4 Evaluate offspring
      3.5 Select survivors for the next generation
```

Parents and/or survivors selection is based on a selective pressure towards fitter (better) individuals.

EA Design

- Representation
- Initialisation
- Recombination
- Mutation
- Parent selection
- Survivor selection
- Termination criteria

- Fitness function
- How to deal with constraints

Feature Selection Problem Formulation

- Design variable:
    - $v \in \{0,1\}^N$, where $v_i = 0$ represents that input feature i is not selected, $v_i = 1$ represents that input feature i is selected, and N is the number of available input features.
- Objective function:
    - Validation error of the model built based on the training set using only the selected features.
- Constraints:
    - None.

What optimisation algorithm would you choose to solve this problem?

EA Design

Evolutionary algorithms are popular choices of algorithms due to their ability to avoid local optima.

- Representation
- Initialisation
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EA Design

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Representation
Initialisation
Recombination
Mutation
Parent selection
Survivor selection
Termination criteria

Fitness function
How to deal with constraints

Summary of Variants of Evolutionary Algorithms

- Representation:
  - Binary strings
  - Integer vectors
  - Floating-point vectors
  - Permutations
  - Matrices
  - Etc
- Crossover for binary, integer and floating point representation
  - 1-Point Crossover
  - Multi-parent diagonal crossover
  - Uniform
- Crossover for floating point representation
  - Intermediate
- Crossover for permutations
  - 1-point crossover for permutations
- Mutation for binary representation
  - Bitwise bit-flipping
- Mutation for integer representation
  - Random reset
  - Creep mutation
- Mutation for floating-point representation
  - Uniform
  - Non-uniform
- Mutation for permutations
  - Swap mutation
- Parents selection:
  - Roulette wheel
  - Tournament selection
- Survival selection:
  - Age-based
    - Generational
  - Fitness-based
    - Delete-worst
    - Elitism

Optimisation Problems May Have Implicit or Explicit Constraints

- Optimisation problem:
  - Maximise / minimise $f(x)$, where x is the design variable
  - Subject to:
    - $g_i(x) \leq 0$, where $i=1,\ldots,n$
      [inequality constraints]
    - $h_j(x) = 0$, where $j=1,\ldots,p$
      [equality constraints]

We will refer to both inequality and equality constraints by $\phi_i(x)$, $i = 1,\ldots,m$, where $m = n + p$ is the total number of constraints.

How to Deal with Constraints?

- Penalty functions.
- Maintaining only feasible solutions based on special representation and genetic operators.

How to Deal with Constraints?

- **Penalty functions** - add a penalty to the fitness of infeasible solutions (or replace the fitness by this penalty).
- Maintaining only feasible solutions based on special representation and genetic operators.

Image from: http://globe-views.com/dcim/dreams/death/death-02.jpg

Death Penalty

Penalty Based on the Level of Infeasibility

$$Q(x) = \begin{cases} 0 & \text{if } x \text{ is feasible} \\ c \cdot \phi(x)^2 & \text{otherwise} \end{cases}$$

Violated constraint (squared)

Very large positive constant

$$Q(x) = \begin{cases} 0 & \text{if } x \text{ is feasible} \\ c \cdot [\phi_1(x)^2 + \phi_2(x)^2 + \ldots + \phi_k(x)^2] & \text{otherwise} \end{cases}$$

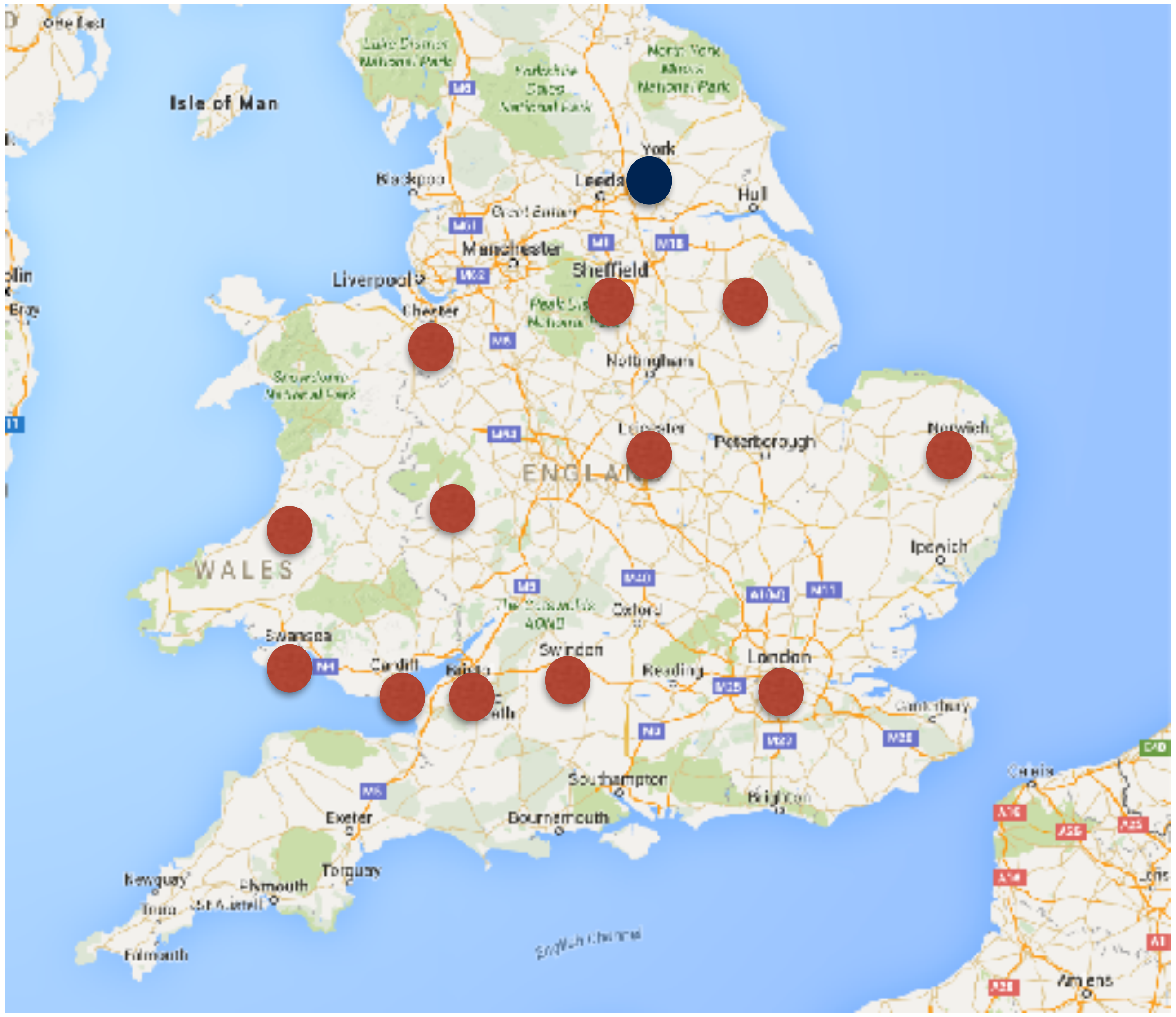
ϕ_i, $1 \leq i \leq k$, are only the violated constraints

How to Deal with Constraints?

- Penalty functions.
- **Maintaining only feasible solutions based on special representation and genetic operators.**

Example: Traveling Salesman Problem

- Traveling Salesman Problem:
  - A salesman must travel passing through *N* cities.
  - Depending on the route the salesman takes, he/she will need to travel longer / shorter distances.



Problem: find a route that minimises traveling distance.

Example: Traveling Salesman Problem

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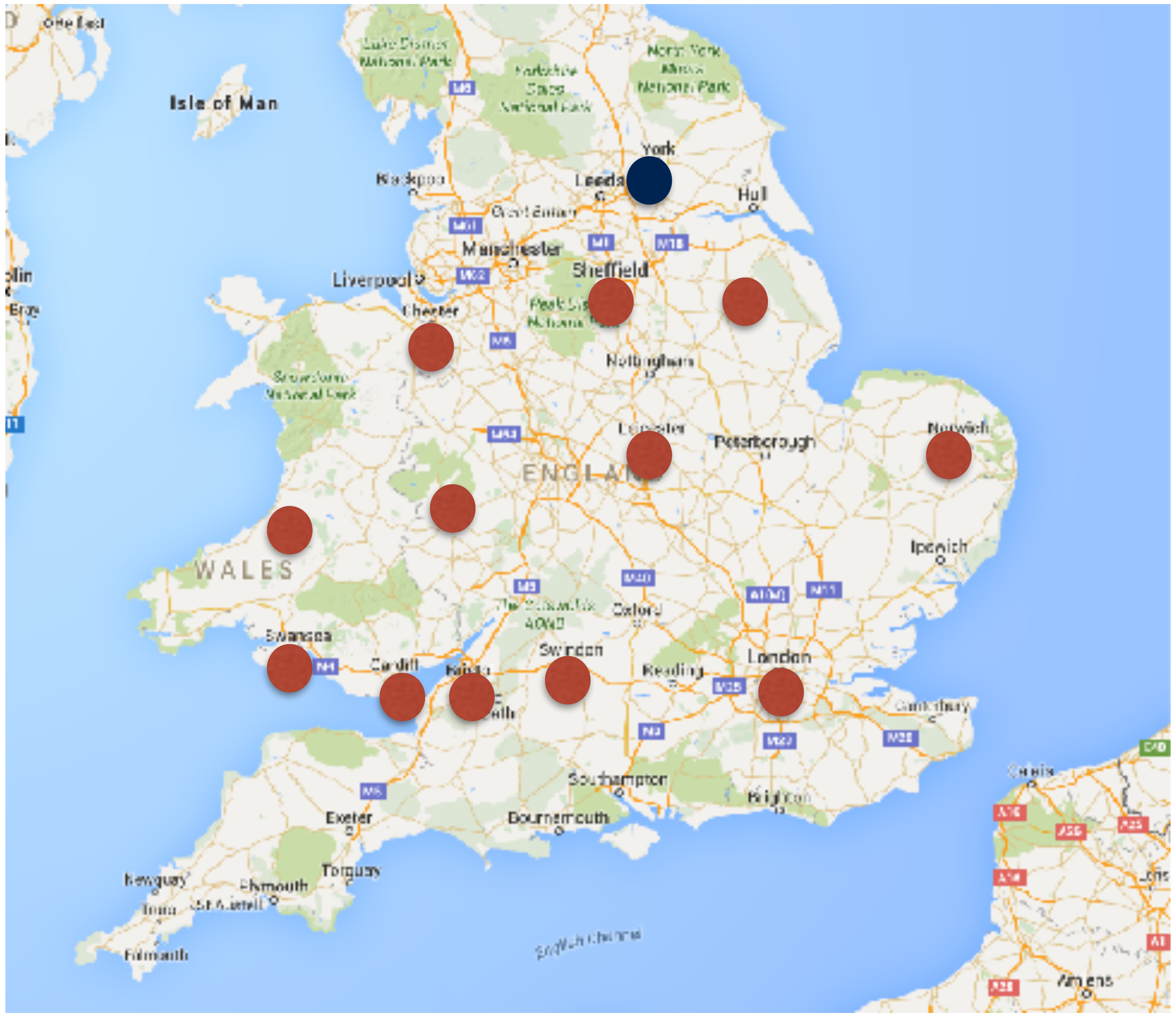


Representation: permutation
Mutation: swap mutation
Crossover: 1-point crossover for permutations

Repair and Decoding

- Repair:
  - Fix the genotype to make it feasible.
  - E.g., remove products from genotype if they exceed the lorry’s maximum supported weight.

- Decoding:
  - Make the solution feasible as you decode it from its genotype to phenotype, without changing the genotype.
  - E.g., normalisation in software project scheduling.

Multi-Objective Optimisation Problems

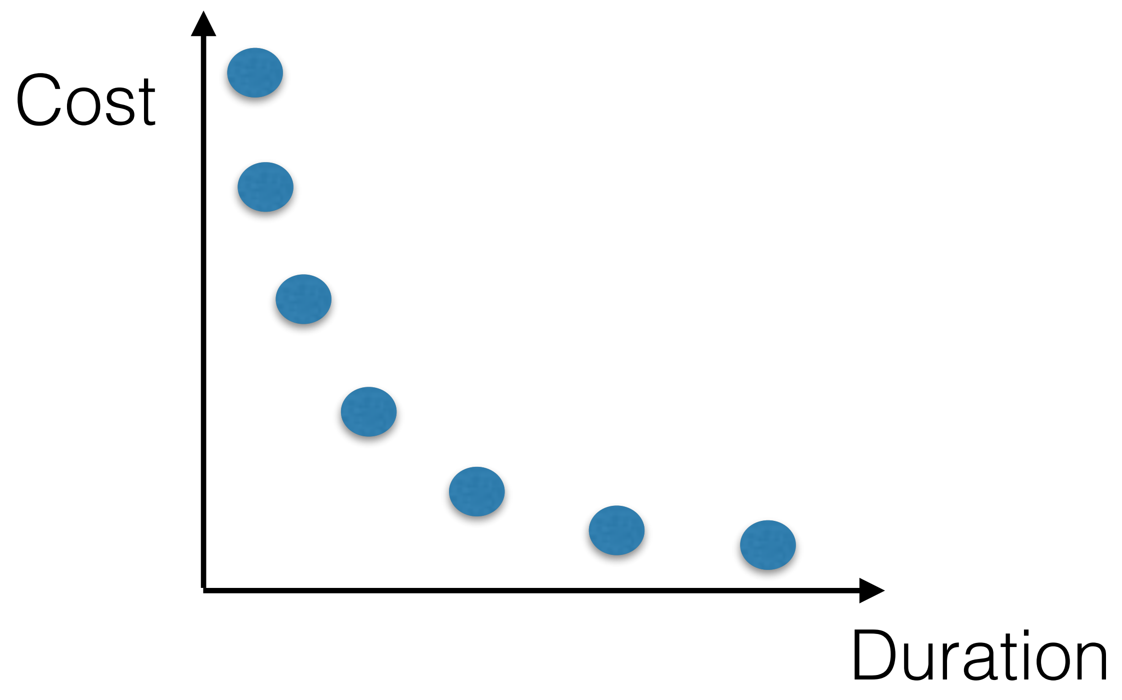
- Real world problems frequently have more than one objective.
- Software project scheduling problem:
    - Cost and duration (to be minimised).

Combining Multiple Objectives into One Objective

- Advantage:
    - Once weights are set, any single-objective optimisation algorithm can be used.
- Disadvantages:
    - Finding suitable weights is not easy before knowing what different trade-offs among objectives may be available.
    - If the different objectives have different scales, weights must reflect not only preferences towards certain objectives, but also treat the different scales.
    - The weights will strongly affect the results.

Multi-Objective Evolutionary Algorithms

- Consider different objectives separately, instead of combining them into a single fitness function.



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