

CO3091 - Computational Intelligence and Software Engineering

Lecture 26



Feature Selection and Revision of Optimisation Problems

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Overview

- The need for feature selection
- How to select input features to use with machine learning approaches
- Revision:
 - Lorry problem
 - Requirements optimisation problem
 - Software project scheduling
 - Software energy optimisation
 - Evolutionary software testing
- Feature selection as an optimisation problem

Module Questionnaire

<https://leicester-surveys.qmihub.co.uk/>

Please complete the module questionnaire for CO3091.

Selecting Input Features for Machine Learning Approaches

- Previous lectures:
 - Practitioners may have some idea about what input features are likely to be related to the output being predicted.
- Software defect prediction:
 - McCabe cyclomatic complexity
 - Halstead complexity measures
 - Lines of code
 - Lines of comments
 - etc.

Selecting Input Features for Machine Learning Approaches

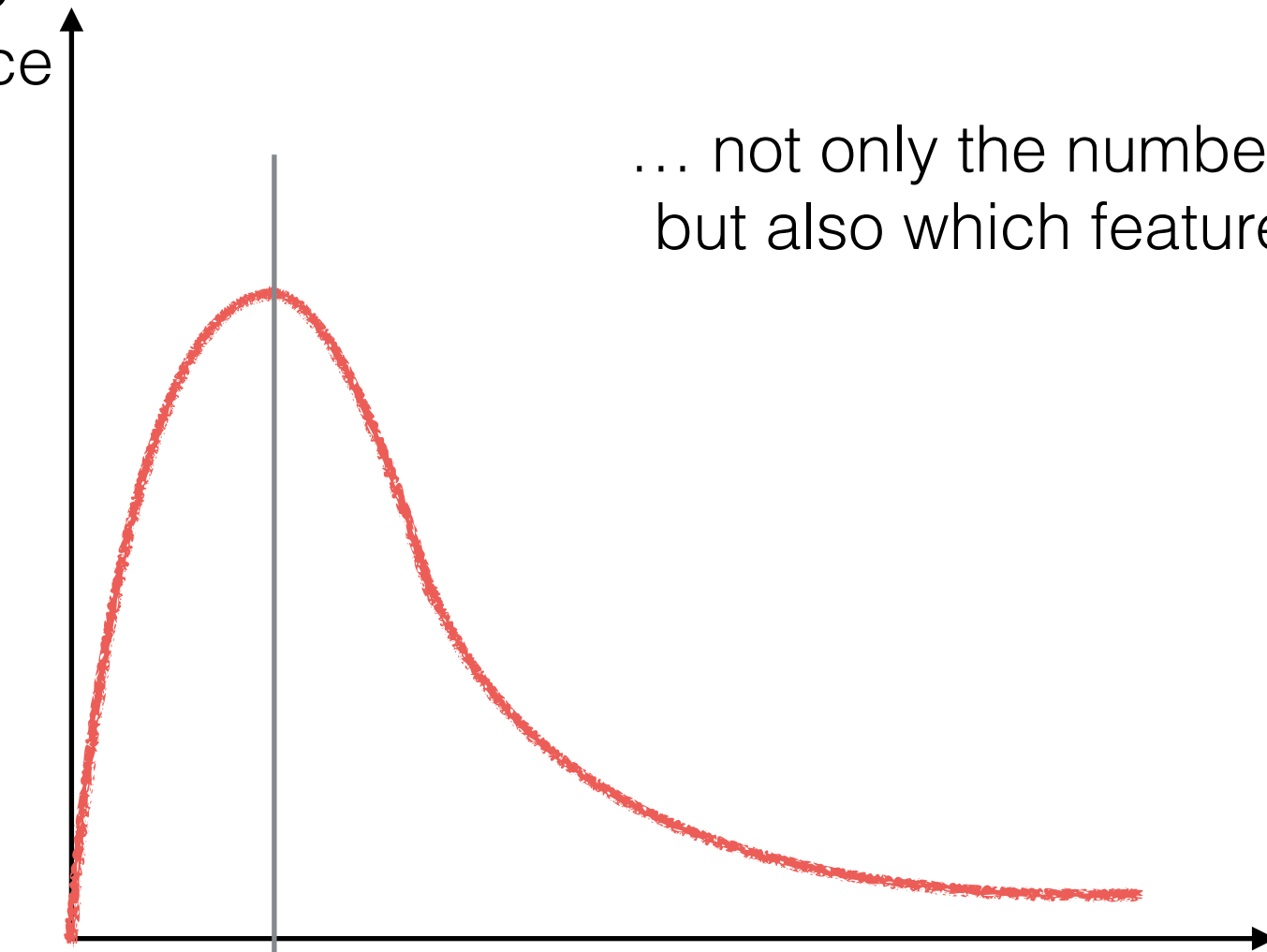
- If we miss including some important input feature, our predictive models may perform poorly.
- **Result:** to avoid missing some important feature, practitioners may suggest many input features, which are not all useful.
 - Is lines of comment really related to defects?
 - Is Halstead Volume needed if we already use LOC?

The Curse of Dimensionality



The Curse of Dimensionality

Predictive performance



... not only the number of features matter, but also which features are being used.

Optimum number of input features

Number of input features (dimensions)

Why Does This Happen?

- As we increase the number of input features (dimensions), we have less and less training data representing certain combinations of input feature values.
 - Certain areas of the input space will be uncovered.
- To cover those input feature values, we would need more training data, which is frequently unavailable.
 - The amount of training data needed often grows exponentially with the number of input features.

Therefore, having too many input features has a similar effect to having too few data, hindering the predictive performance of machine learning approaches.

Why Does This Happen? (cont.)

- Instances that are similar to each other on some input features may be very different on others.

1	1	0	0	0	0	0	0
1	1	0	0	1	1	1	1

- In the example, if the last 4 input features are irrelevant to the problem, they will deceive the algorithm into thinking that the two instances above are dissimilar when they are actually very similar.
- This can affect some machine learning approaches.

Select only the best features

“Make things as simple as possible,
but no simpler”

— Albert Einstein

Don't miss any important feature

How to Select Features Among a Set of Potential Features?

- Filter feature selection methods:
 - Light, can be applied before building a predictive model.
 - E.g.: correlation-based feature selection method (CFS).
- Wrapper feature selection methods.
 - Formulate feature selection as an optimisation problem.
 - We want to select the features that lead to better predictive performance.
 - Heavier, but possibly leads to better predictive performance than filter methods.

We can use optimisation algorithms!

Breakpoint!

Formulation of Optimisation Problems

- **Design variables** represent a solution.
- Design variables define the **search space** of candidate solutions.
- [Optional] Solutions must satisfy certain **constraints**.
- **Objective function** defines our goal.
 - Can be used to evaluate the quality of solutions.
 - Function to be optimised (maximised or minimised).

Lorry Problem

- Consider the following problem:
 - You need to load a lorry with products. **The maximum total weight of products that the lorry can stand is W .**
 - You have N products that can be loaded, and each product i has a weight w_i , and a profit p_i .

Problem: **decide** which products to load so as to **maximise** the total profit of loaded products.



Image from: <http://www.wingstransport.com/cms-files/hpSlide-1357902701.png>

Lorry Problem Formulation

- Design variable:
 - $v \in \{0,1\}^N$, where $v_i = 0$ represents that product i is not loaded, $v_i = 1$ represents that product i is loaded, and N is the number of products.

- Objective function:

$$f(v) = \sum_{i=1}^N v_i p_i \quad (\text{to be maximised}).$$

0 if product i is not loaded
1 otherwise
Total profit of loaded products

- Constraint:

$$\sum_{i=1}^N v_i w_i \leq W$$

Total weight of loaded products



Requirements Selection

- As requirements have a **cost**, we may need to select a subset of all possible requirements to implement, so that the project will:
 - be within **budget** or
 - have **lower cost**.
- We need to decide which possible requirements to implement, considering (potentially among others):
 - their **cost**,
 - their **value** from different stakeholders perspectives,
 - the **importance of the stakeholder** who wants the requirement.

Requirements Selection Problem Formulation

Decision variable: $\mathbf{x} = \{0, 1\}^n$

n is the number of requirements

$x_j = 0$ if requirement r_j is not included

$x_j = 1$ if requirement r_j is included

Objective: Maximise $\sum_{j=1}^n \text{score}(r_j) x_j$

Constraint: $\sum_{j=1}^n \text{cost}(r_j) x_j \leq C$

Score reflects value of requirement from each stakeholder point of view and the importance of the stakeholder.

total cost of selected requirements

Software Project Scheduling

Setting: assume we are given

- n employees e_1, \dots, e_n with salaries sal_i and sets of skills $skill_i$;
- m tasks t_1, \dots, t_m with required efforts $reqEff_j$ and sets of required skills $reqSk_j$;
- a task precedence graph (TPG).

Problem: allocate employees to tasks so as to:

- **minimise cost** (total salaries paid) and
- **minimise duration** (completion time).

Constraints:

- **team must have required skills and**
- **no overwork.**

Formulation — Design Variable

- Design variable:
 - \mathbf{x}' = Matrix of employees by tasks.
 - Entries $x'_{i,j}$ represent the dedication of employee i to task j .
 - Dedication = percentage of the employee's time dedicated to the task.
 - Values that each entry $x'_{i,j}$ could assume depend on the granularity k of the problem.

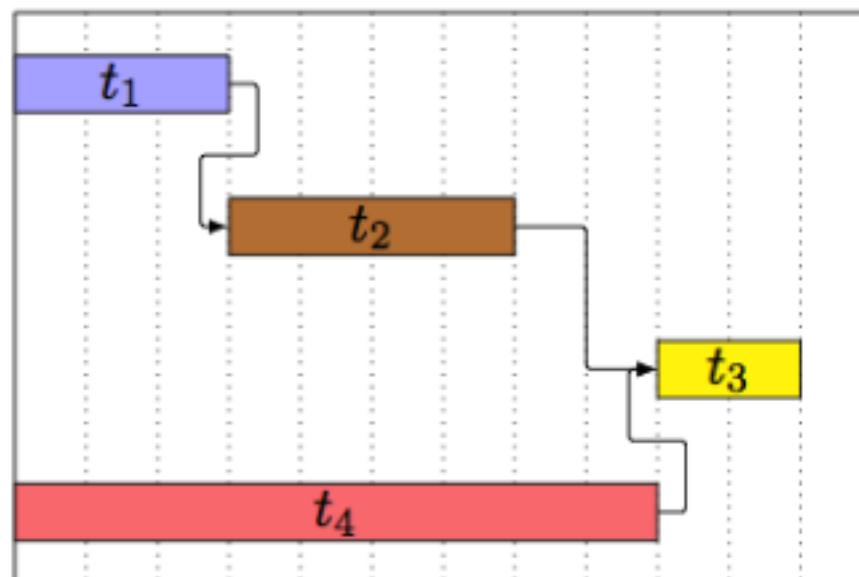
\mathbf{x}'	\mathbf{t}_1	\mathbf{t}_2	...	\mathbf{t}_m
\mathbf{e}_1	$X'_{1,1}$	$X'_{1,2}$...	$X'_{1,m}$
\mathbf{e}_2	$X'_{2,1}$	$X'_{2,2}$...	$X'_{2,m}$
...
\mathbf{e}_n	$X'_{n,1}$	$X'_{n,2}$...	$X'_{n,m}$

$$x'_{i,j} \in \left\{ \frac{0}{k}, \frac{1}{k}, \frac{2}{k}, \dots, \frac{k}{k} \right\}$$

Calculating Cost and Duration

x'	t_1	t_2	t_3	t_4
e_1	$X'_{1,1}$	$X'_{1,2}$	$X'_{1,3}$	$X'_{1,4}$
e_2	$X'_{2,1}$	$X'_{2,2}$	$X'_{2,3}$	$X'_{2,4}$
e_3	$X'_{3,1}$	$X'_{3,2}$	$X'_{3,3}$	$X'_{3,4}$

↓ + TPG, tasks required efforts

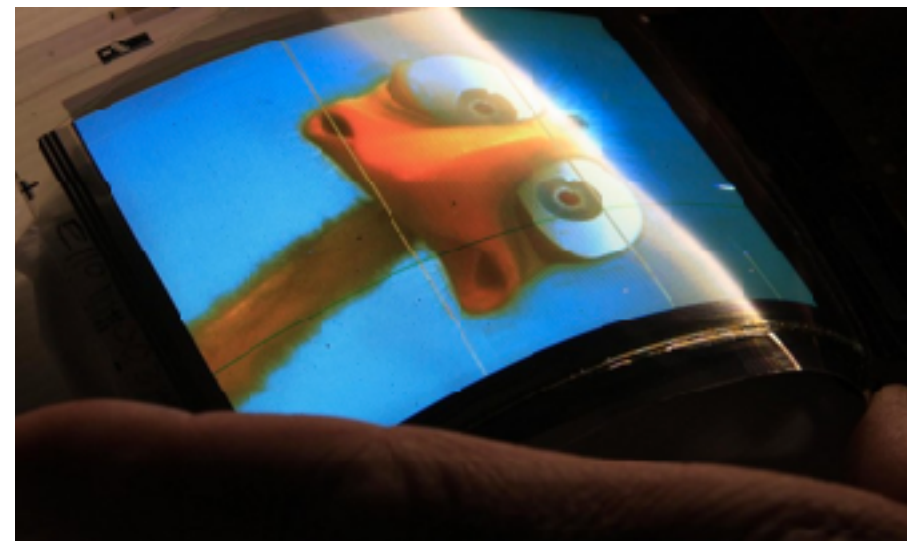


+ salaries
→

Cost and duration

Software Energy Optimisation

- Mobile apps have widespread use.
- Mobile apps consume energy (battery).
- Studies show that battery consumption is one of the key factors considered by users when choosing a mobile app.
- In order to reduce energy consumption of OLED screens, one can optimise the **colours used by the GUI**.



Software Energy Optimisation — Formulation

- Consider an initial GUI design with its colours.
- **Design variables:** RGB colour of each pixel.
- **Objectives:**
 - Minimise power consumption on OLED displays.
 - Power consumption of a GUI depends on the power consumption of its screens and the amount of time users spend on different screens.
 - Power consumption of a screen depends on the power consumption of its pixels.
 - Power consumption of a pixel is the sum of the power consumptions of its RGB colour components.
 - Maximise contrasts between adjacent GUI elements.
 - Minimise difference with respect to the original GUI design.
- **Constraints:**
 - Adjacent elements of the GUI should not have the same colour or colours with too low contrast between them.

Evolutionary Software Testing

- Software testing is an essential component for software development success.
- Software testing is one of the most expensive tasks in the software development process.
- **Test suite:** set of **test cases**, each consisting of a sequence of inputs and expected outputs from the program.
- Challenging for large and complex software.
- A good test suite should **exercise the code well** and be **fast to run**.

Evolutionary Software Testing — Formulation

- **Design variable:** list with a given number of input sequences.
 - Design variable can be seen as a **test suite**.
 - Each input sequence is a test case with an expected “output” of “**no crash**”.
 - **Examples of inputs for Android:** Touch, Motion, Rotation, Trackball, PinchZoom, Flip, Nav, MajorNav, AppSwitch, SysOp, enter text, or clicks on widgets.
- **Objectives:**
 - Maximise coverage.
 - Minimise length of test cases.
 - Maximise number of crashes found.
- **Constraints:**
 - N/A

Back to Feature Selection



Image from: https://images-na.ssl-images-amazon.com/images/S/sgp-catalog-images/region_US/nbcu-61101081-Full-Image_GalleryBackground-en-US-1484000598187._RI_SX940_.jpg

Wrapper Methods

- Formulate feature selection as an optimisation problem.
- We want to **select** the features that lead to **better predictive performance**.

Feature Selection Problem Formulation

- Design variable: ?
- Objectives: ?
- Constraints: ?

Feature Selection Problem Formulation

- Design variable:
 - $v \in \{0, 1\}^N$, where $v_i = 0$ represents that input feature i is not selected, $v_i = 1$ represents that input feature i is selected, and N is the number of available input features.
- Objective function:
 - Predictive performance of model created using the selected features.
 - Build a predictive model using a machine learning algorithm and the training set using the selected input features.
 - Evaluate the predictive performance of this predictive model.

Examples of Evaluation Functions Using a Known Data Set

- **Classification error:**

- Given a data set D with examples (\mathbf{x}_i, y_i) , $1 \leq i \leq m$.
- The actual output (target) for \mathbf{x}_i is y_i .
- The prediction given by a classification model to \mathbf{x}_i is y_i' .
- y_i and y_i' are categorical values.

$$\text{Classification error} = \frac{1}{m} \sum_{i=1}^m (y_i \neq y_i')$$

- **Classification accuracy:**

$$\text{Classification accuracy} = 1 - \text{classification error}$$

Examples of Evaluation Functions Using a Known Data Set

- Mean Squared Error (MSE):

- Given a data set D with examples (\mathbf{x}_i, y_i) , $1 \leq i \leq m$.
- The actual output (target) for \mathbf{x}_i is y_i .
- The prediction given by a regression model to \mathbf{x}_i is y_i' .
- y_i and y_i' are numerical values.

$$\text{MSE} = \frac{1}{m} \sum_{i=1}^m (y_i - y_i')^2$$

- Root Mean Squared Error (RMSE):

$$\text{RMSE} = \sqrt{\text{MSE}}$$



Difference between optimisation and machine learning from problem perspective:
in machine learning we wish to create models with good generalisation ability.

Which Data To Use For Computing the Predictive Performance?

- We had three different types of data sets:
 - **Training set:**
 - Used by the machine learning approach to learn a model.
 - **Validation set:**
 - Used to choose between different machine learning approaches (or parameters for the approaches). It estimates the error on data unseen at the time of building the model.
 - **Test set:**
 - Separate data set used neither for training nor for validation. It can be used to give an idea of how well the model will perform / is performing in practice, i.e., how good the generalisation to future unseen data is likely to be.

Feature Selection Problem Formulation

- Design variable:
 - $v \in \{0,1\}^N$, where $v_i = 0$ represents that input feature i is not selected, $v_i = 1$ represents that input feature i is selected, and N is the number of available input features.
- Objective function:
 - Validation error of the model built based on the training set using only the selected features.
- Constraints:
 - None.

Now that the problem is formulated, it is possible to design an optimisation algorithm to solve it!

Overview

Optimisation problems:

- Feature selection
- Lorry problem
- Requirements optimisation problem
- Software project scheduling
- Software energy optimisation
- Software test case generation

Next lecture:

- Optimisation algorithms