

# CO3091 - Computational Intelligence and Software Engineering

## Lecture 24



# Ensembles of Learning Machines

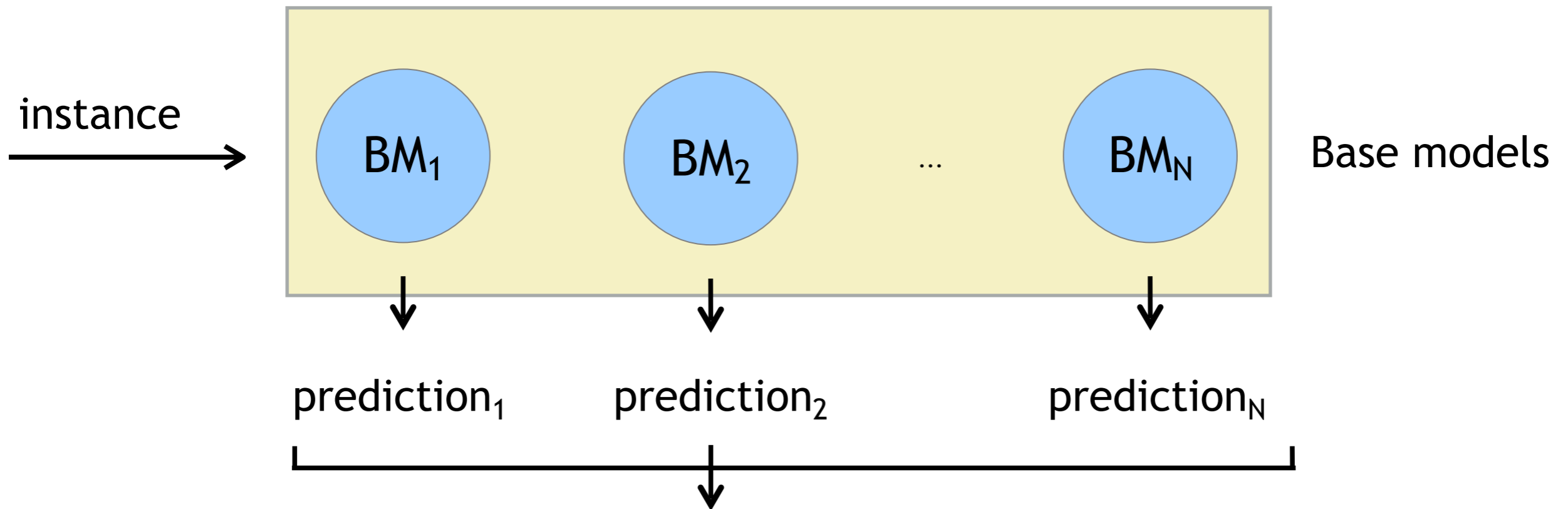
Leandro L. Minku

# Overview

- So far in the machine learning part of the module...
  - k-Nearest Neighbours.
  - Naive Bayes.
  - Decision Trees.
- Today:
  - Combining learners into ensembles in order to improve predictive performance.
  - When do ensembles work?
  - Bagging ensembles.
  - Software effort estimation.

# What are Ensembles?

Ensembles are sets of learning machines grouped together with the aim of reducing error / increasing accuracy.

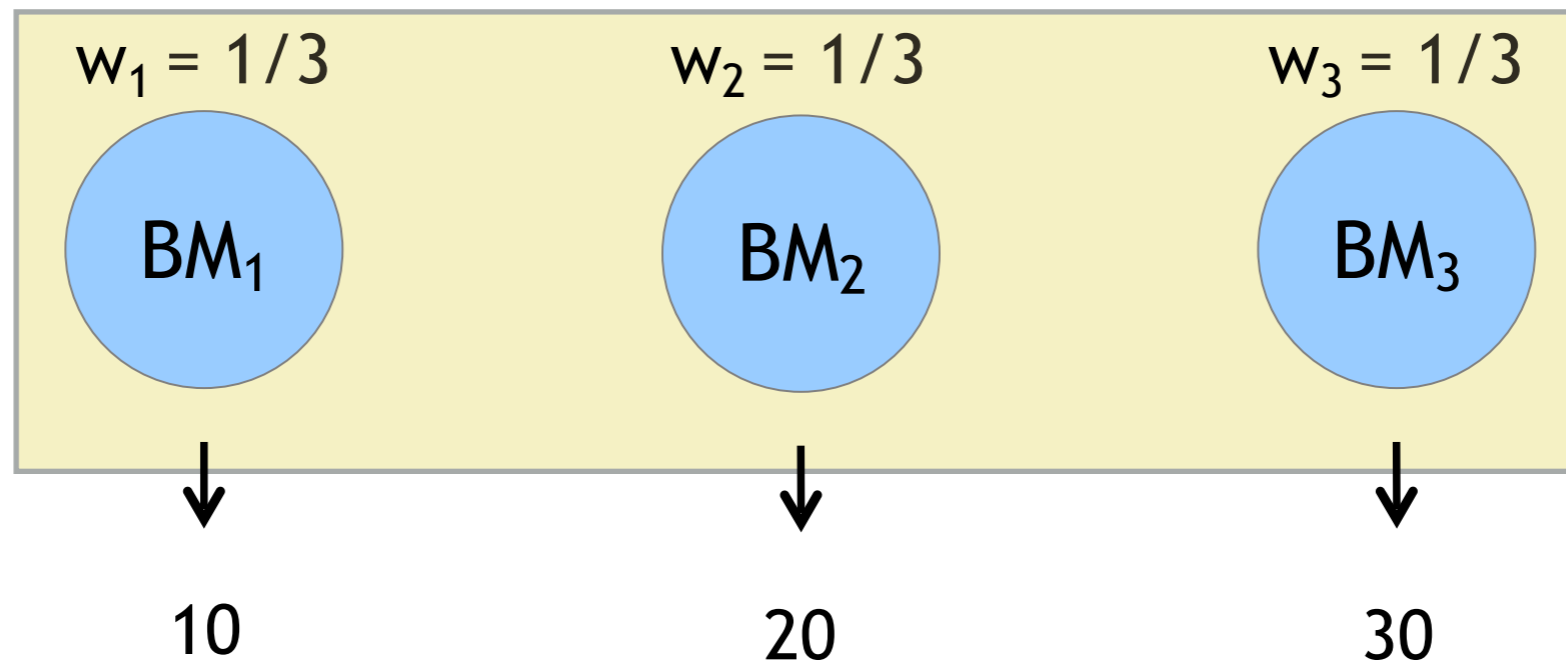


E.g.: ensemble prediction =  $\sum w_i \text{prediction}_i$

E.g.: ensemble prediction = weighted majority vote among  $\text{prediction}_i$

★  $\sum w_i = 1$

# Example of Predictions

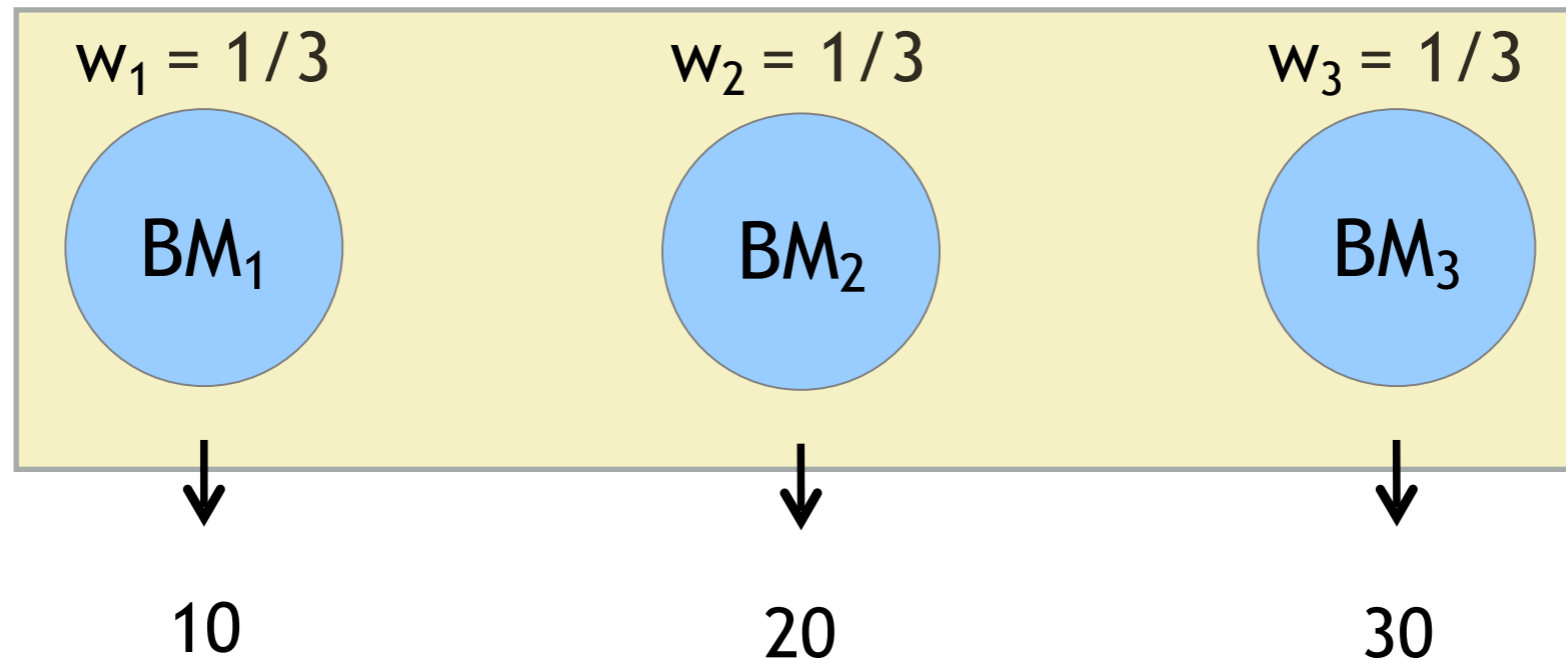


ensemble prediction =  $\sum w_i \text{prediction}_i$

ensemble prediction = simple average of  $\text{prediction}_i$

ensemble prediction = ?

# Example of Predictions

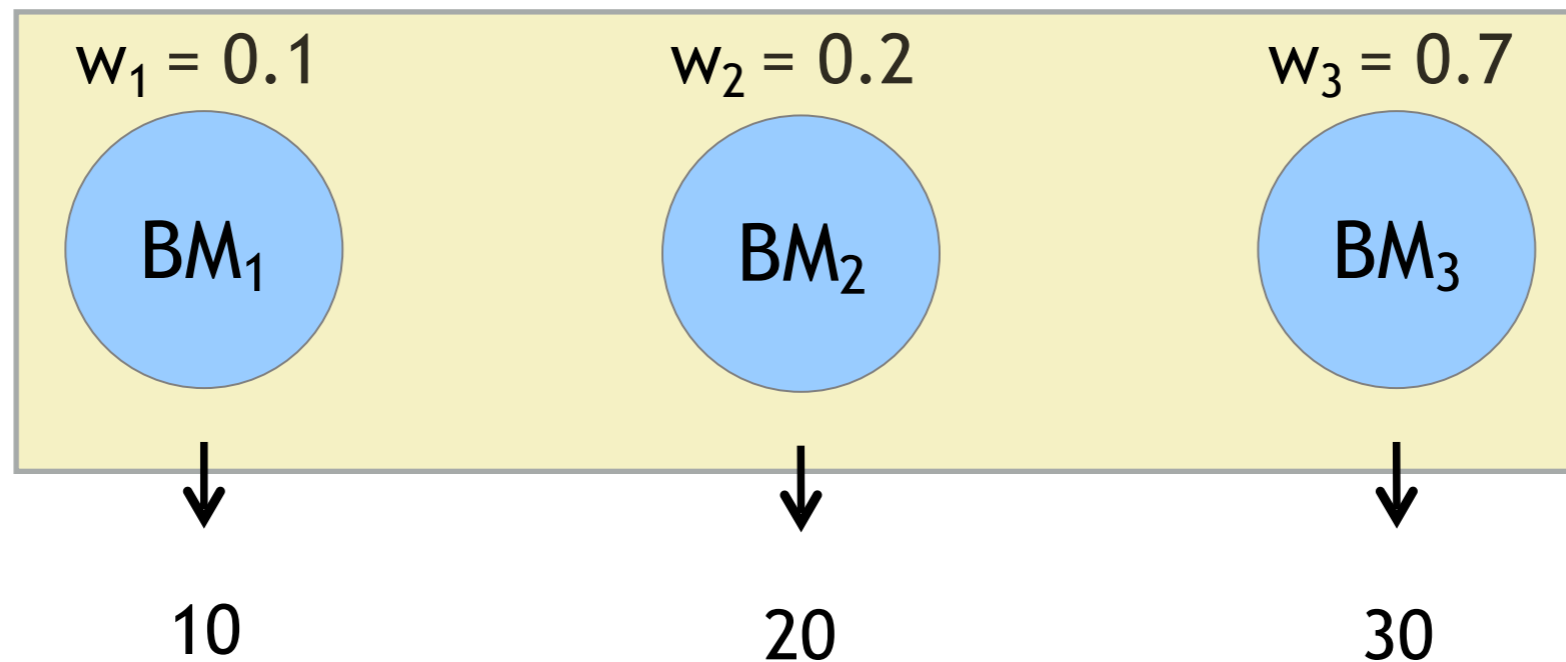


ensemble prediction =  $\sum w_i \text{prediction}_i$

ensemble prediction = simple average of  $\text{prediction}_i$

ensemble prediction = 20

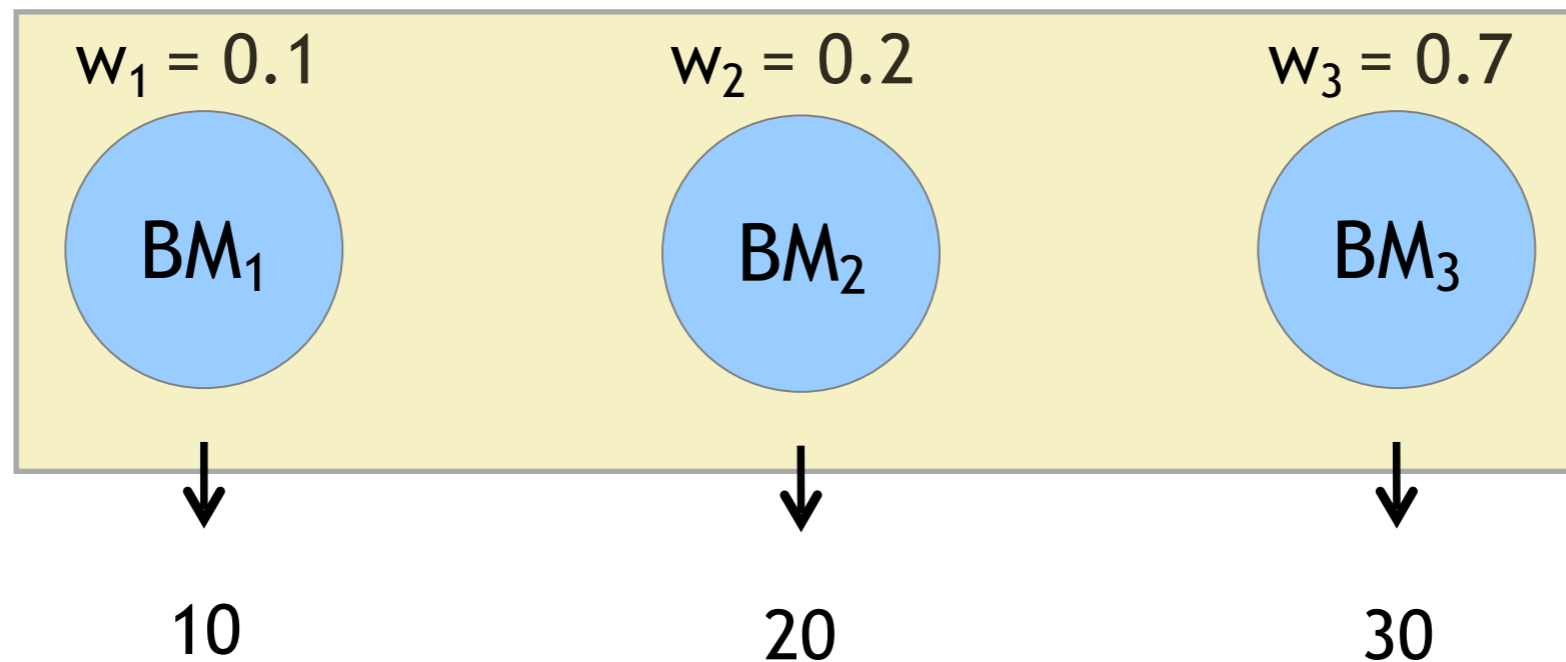
# Example of Predictions



ensemble prediction =  $\sum w_i \text{prediction}_i$

ensemble prediction = ?

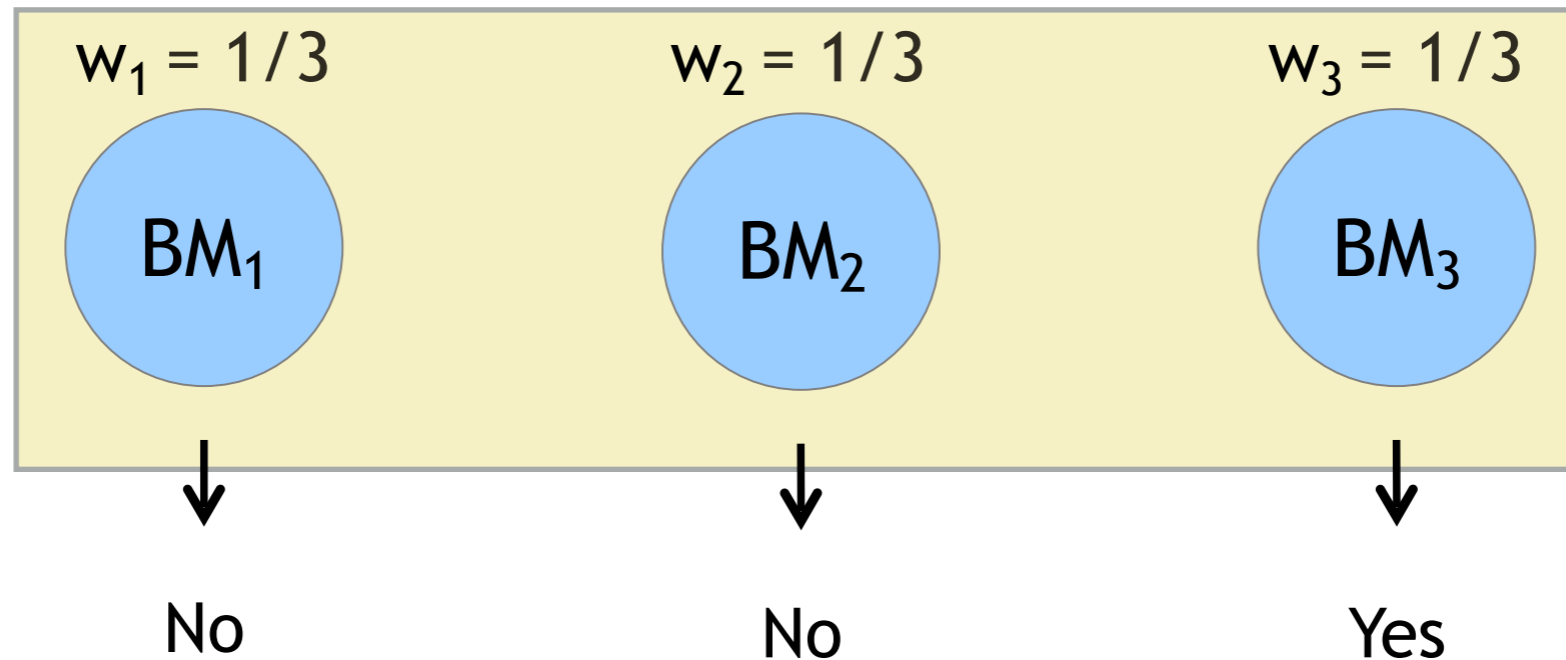
# Example of Predictions



ensemble prediction =  $\sum w_i \text{prediction}_i$

$$\text{ensemble prediction} = 0.1 * 10 + 0.2 * 20 + 0.7 * 30 = 1 + 4 + 21 = 26$$

# Example of Predictions



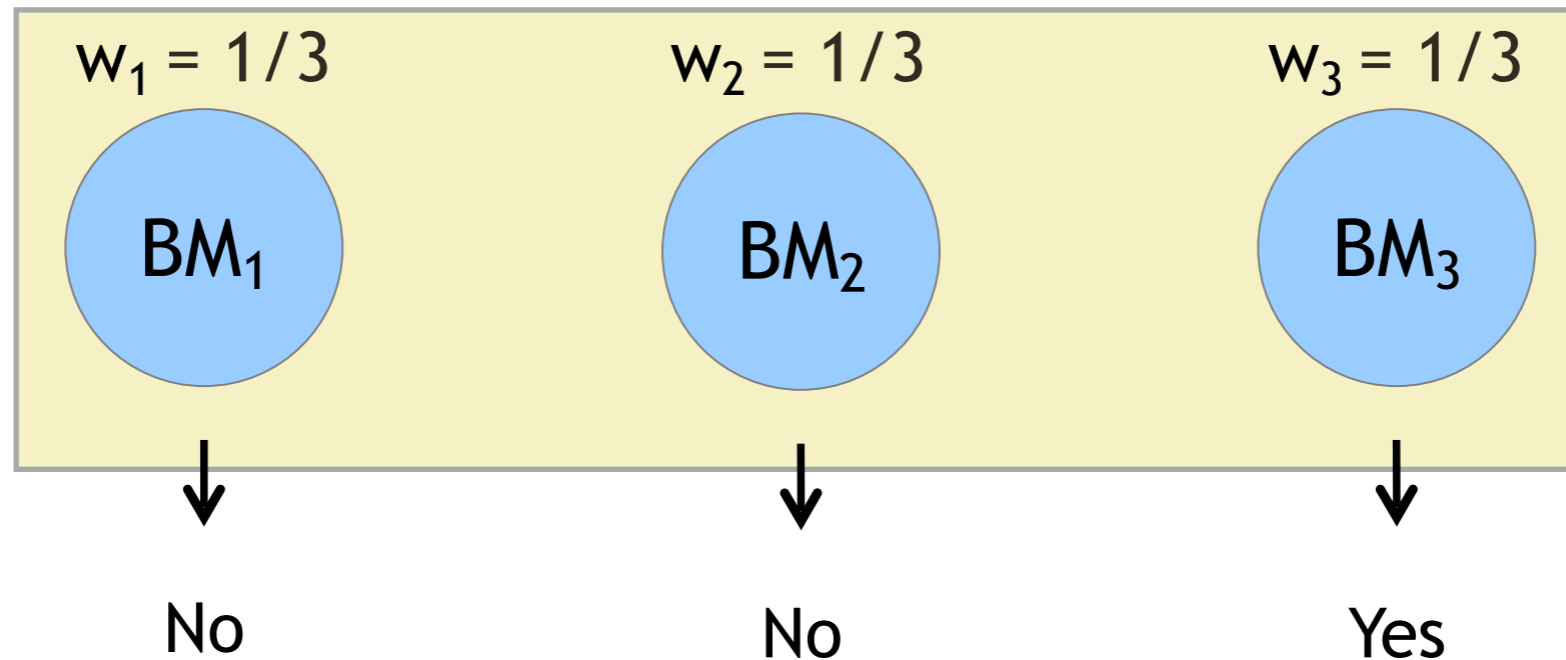
ensemble prediction = weighted majority vote among prediction<sub>i</sub>

ensemble prediction = majority vote among prediction<sub>i</sub>

ensemble prediction = ?



# Example of Predictions

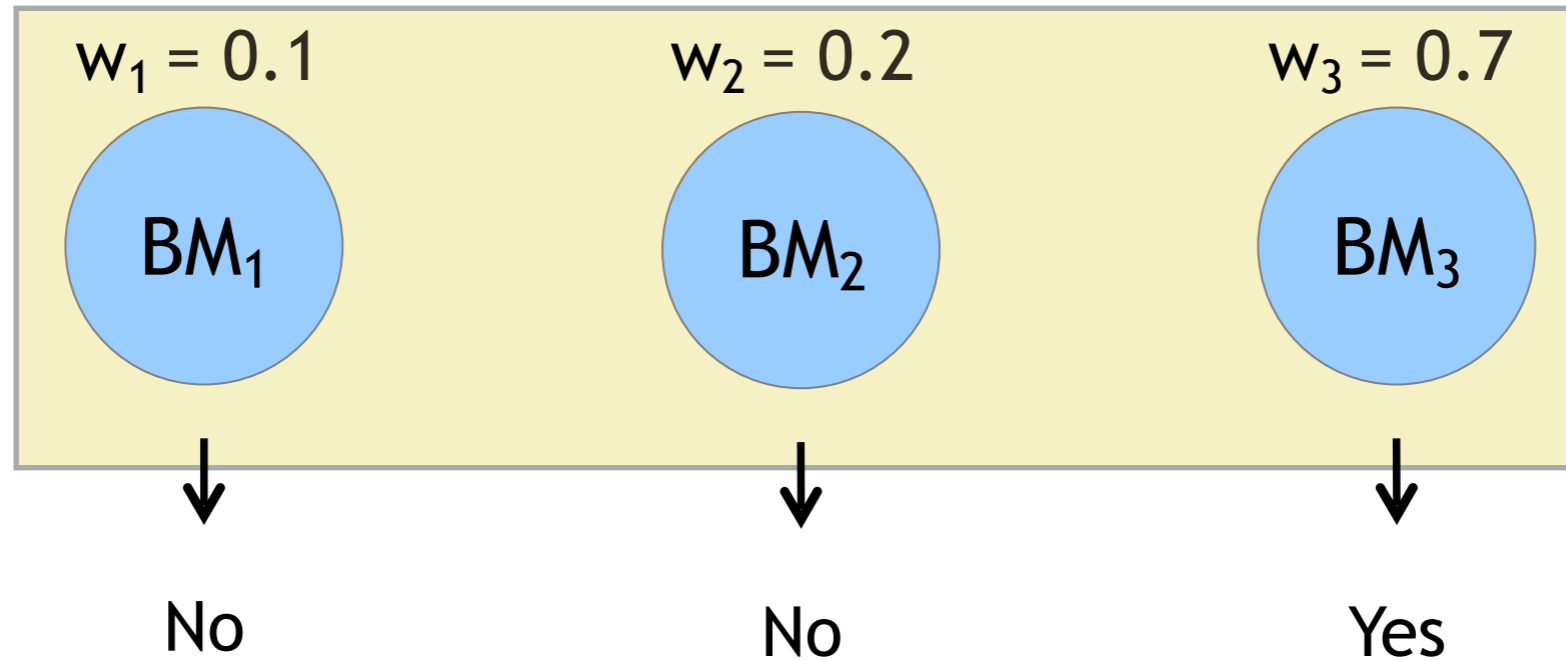


ensemble prediction = weighted majority vote among prediction<sub>i</sub>

ensemble prediction = majority vote among prediction<sub>i</sub>

ensemble prediction = No

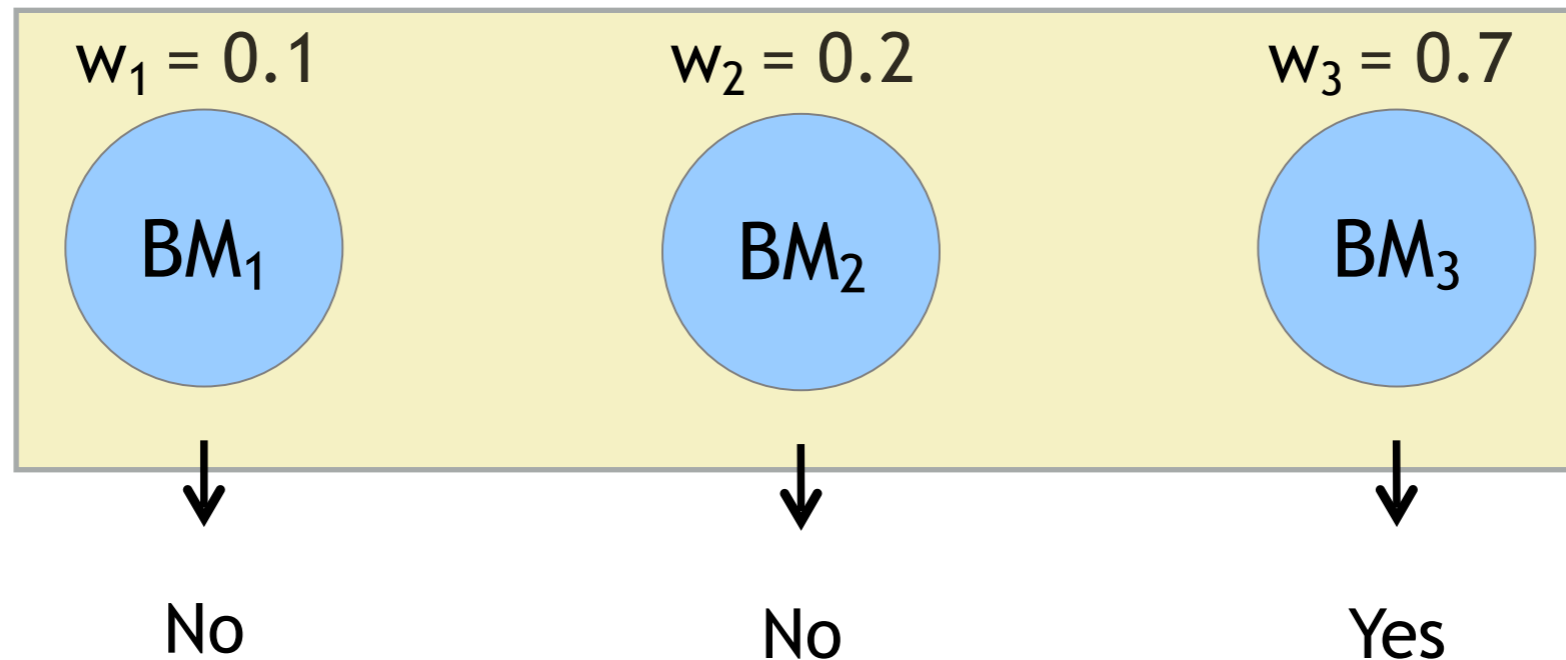
# Example of Predictions



ensemble prediction = weighted majority vote among prediction<sub>i</sub>

ensemble prediction = ?

# Example of Predictions



ensemble prediction = weighted majority vote among prediction<sub>i</sub>

Votes for No:  $0.1 + 0.2 = 0.3$

Votes for Yes:  $0.7$

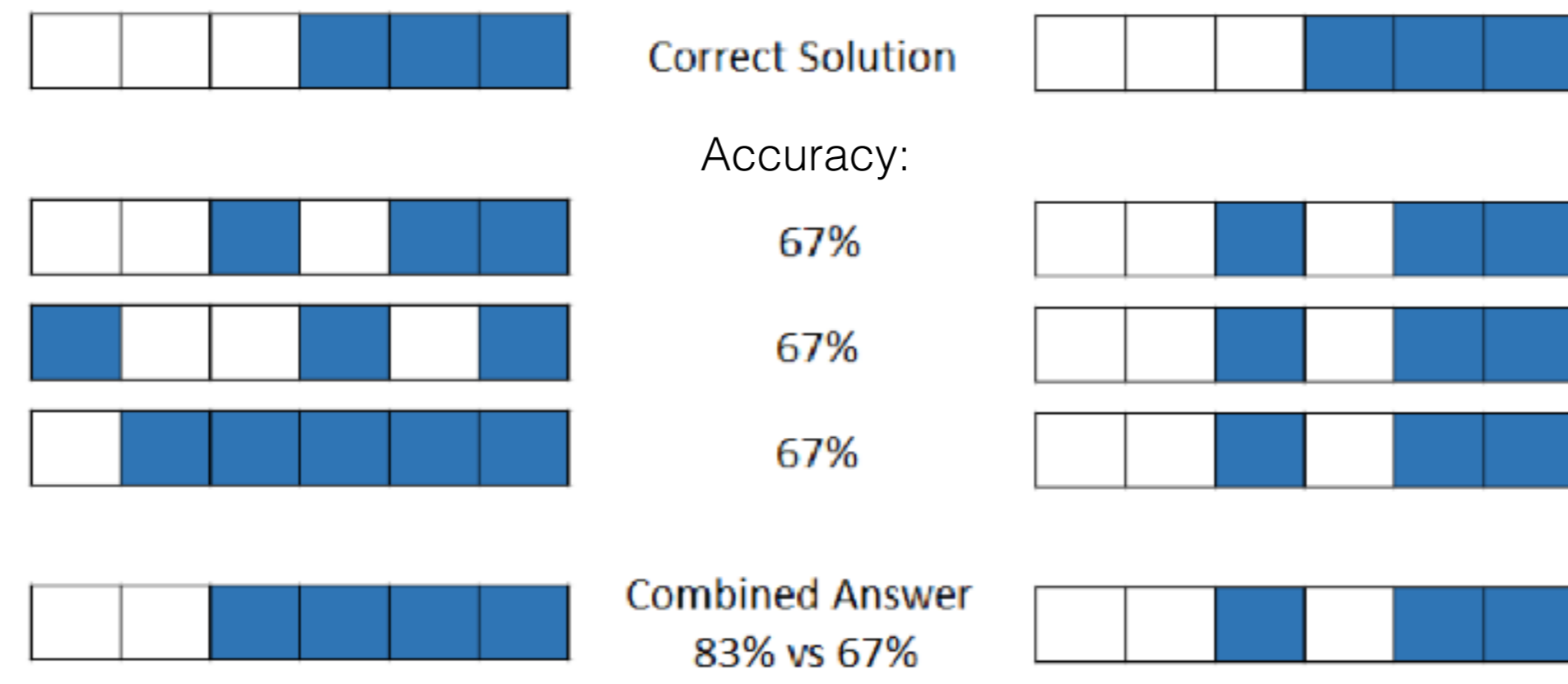
ensemble prediction = **Yes**

# When Do Ensembles Work Well?

- Base models should be both accurate and diverse.
  - **Accuracy:**
    - If base models make too many mistakes, the whole ensemble prediction will also make too many mistakes.
  - **Diversity:** models are diverse if they make different mistakes.
    - If all base models make the same mistakes, the whole ensemble prediction will make the same mistakes as the base models.

# Intuition for Classification Problems

**Intuition:** correct predictions given by some models compensate for the incorrect predictions given by the other models.



# Intuition for Regression Problems



[YouTube Video posted by tim6: <https://youtu.be/iOucwX7Z1HU>]

# Intuition for Regression Problems

**Intuition:** overestimations are compensated by underestimations if the base models are diverse enough.

Diversity is necessary both for classification and for regression problems!



# Ensemble Approaches

Different ensemble learning algorithms can be seen as different approaches to generate accurate and diverse base models.

# Bagging (Bootstrap Aggregating)

- Bootstrap sampling:
  - Sample  $|D|$  examples from a training set of size  $|D|$  **uniformly** with **replacement**.
  - **Uniformly** = all examples have the same chance to be selected.
  - **With replacement** = allow an example to be selected more than once to be part of the sample.

Project	Size	Team Expertise	Programming Language	Effort
P1	100	High	Java	80
P2	200	Normal	C++	220
P3	150	High	Python	100
P4	500	Normal	Java	600
P5	550	Normal	C++	700

Training Set With Size  $|D| = 5$

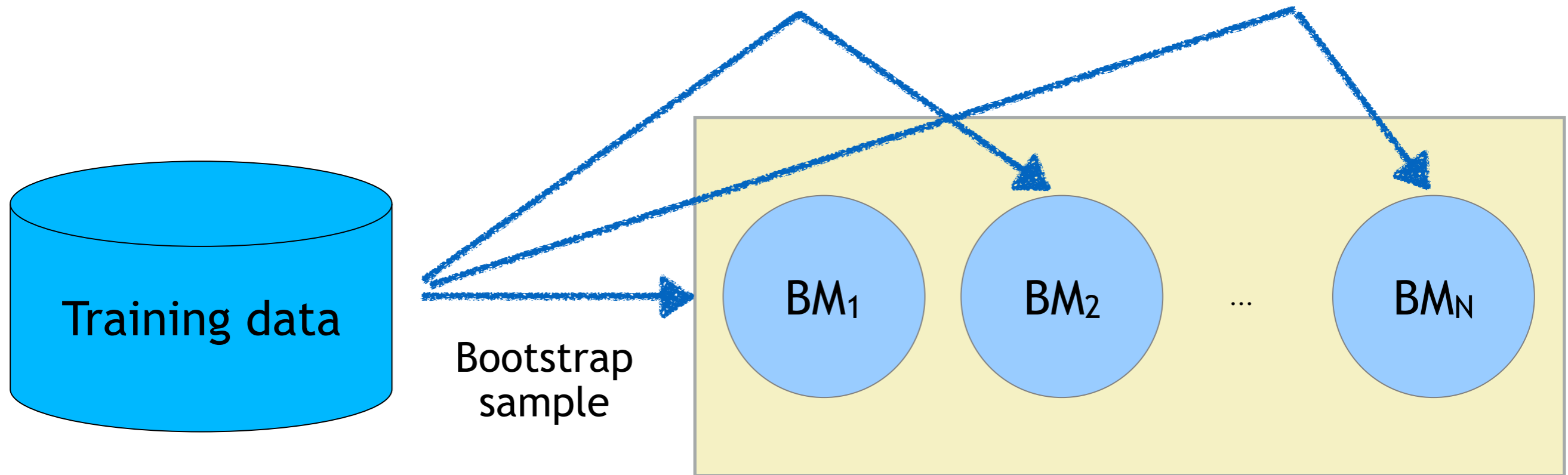
Project	Size	Team Expertise	Programming Language	Effort
P2	200	Normal	C++	220
P5	550	Normal	C++	700
P1	100	High	Java	80
P5	550	Normal	C++	700
P4	500	Normal	Java	600

Sample from Training Set With Size  $|D| = 5$

# Bagging (Bootstrap Aggregating)

- Bootstrap sampling:
  - If you use bootstrap sampling to a sample data set  $D'$  with size  $|D|$  based on an original data set  $D$  of size  $|D|$ , it is expected that  $D'$  will contain 63.2% of the unique examples from  $D$ .
  - If you repeat bootstrap sampling several times, you are likely to get several different sample data sets.
  - Different samples of the training set will normally lead to different base models.

# Bagging (Bootstrap Aggregating)



# Pseudocode

Bagging (ensemble size  $N$ , training set  $D$ , base learning algorithm)

ensemble = {}

For  $i=1$  to  $N$ ,

$D' \leftarrow$  sample  $|D|$  examples from  $D$  uniformly with replacement (bootstrap sample)

Build base model  $BM_i$  using base learning algorithm with  $D'$

ensemble = ensemble  $\cup$   $\{BM_i\}$

Return ensemble

# Bagging Ensemble Predictions

- The ensemble predictions are:
  - **Regression problems:** simple average of the predictions given by the base models.
  - **Classification problems:** majority vote of the predictions given by the base models.

# When and Why Bagging Works?

- Success of bagging depends on base learning algorithm being **good but unstable**.
  - **Good**: creates accurate base models.
  - **Unstable**: small change in the training sample can result in a large change in the predictions given by the resulting base model.
    - This means that enough **diversity** would be generated.
- If a stable base learning algorithm is used, bagging can even worsen the predictive accuracy.

# Stable / Unstable Based Learning Algorithms

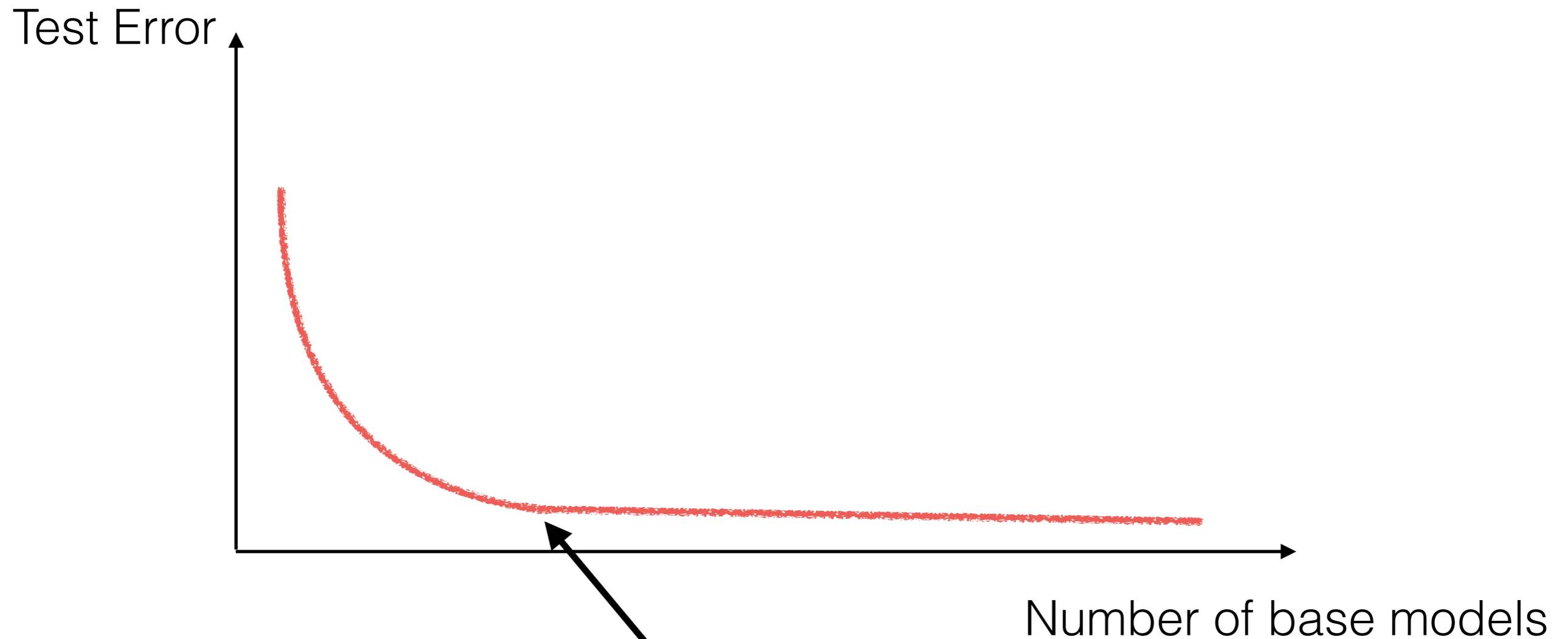
- Example of unstable base learning algorithm: [decision trees](#).
- Example of stable base learning algorithm: [k-NN](#).



# What Ensemble Size Should We Use?

- Early work suggested that ensembles with as few as 10 base models were adequate to sufficiently reduce the test error.
- However, later on it was found that test error can be further reduced even after 10 base models have been added.
- Ensemble size is a parameter of ensemble approaches.
  - The best ensemble size will depend on the data.
  - Too few base models will not be enough to reduce the test error.
  - Too many will waste resources.

# Typical Test Error Curve



In Bagging, this point may be  $\sim 10\text{--}15$  base models for classification problems

# Software Effort Estimation

- Estimation of the effort required to develop a software project.
  - Effort is measured in person-hours, person-months, etc.
- Based on features such as programming language, team expertise, estimated size, development type, required reliability, etc.
- Main factor influencing project cost.
- Overestimation vs underestimation.

# Machine Learning for Software Effort Estimation

- Software effort estimation is difficult to perform by humans.
  - Affected by irrelevant features.
  - Lack of improvement in the predictions over time.
- Machine learning can help.

$x_1 =$ programming language	$x_2 =$ team expertise	$x_3 =$ estimated size	...	$y =$ required effort
Java	low	1000	...	10 p-month
C++	medium	2000	...	20 p-month
Java	high	2000	...	8 p-month
...	...	...	...	...

# Machine Learning for Software Effort Estimation

- **k-NN:**
  - Can be intuitive for practitioners, helping them to find completed projects that are most similar to the new project to be developed.
  - Offer competitive accuracy in comparison to other approaches, but performs poorly for some companies.
- **Decision trees:**
  - Can be visualised.
  - Frequently among the best machine learning approaches for software effort estimation.
- **Bagging ensembles of decision trees.**
  - Can improve accuracy further with respect to decision trees.
  - Difficult to visualise.
- **Naive Bayes:**
  - Usually doesn't work well for regression problems.

# Further Reading

Menzies et al.

Sharing Data and Models in Software Engineering

Elsevier, 2014

Chapter 20 (Ensembles of Learning Machines) until section 20.3

<http://readinglists.le.ac.uk/lists/D888DC7C-0042-C4A3-5673-2DF8E4DFE225.html>

David Opitz

Ensemble Size

<https://www.cs.cmu.edu/afs/cs/project/jair/pub/volume11/opitz99a-html/node10.html>